Uitwerkingen extra opgaven hoofdstuk 4 Limieten en differentiaalrekening

1.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x+1}{3x-1} = \frac{2}{3} \text{(via een tabel te bepalen)}$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} \frac{2x+1}{3x-1} = \frac{2}{3} \text{(via een tabel te bepalen)}$$

Dit betekent dat de grafiek van f zowel naar links als naar rechts een horizontale asymptoot heeft. De vergelijking van deze asymptoot is in beide gevallen $x = \frac{2}{3}$.

2.

a. $$\lim_{x\to-3} \frac{x-3}{x^2+9} = \frac{-6}{9} = -\frac{2}{3}$$

b. $$\lim_{x\to-3} \frac{x-3}{(2x+6)^2} = \lim_{x\to-3} \left(\frac{-6}{0^2}\right) = -\infty$$

c. $$\lim_{x\to-3} \frac{x-3}{x^2-9} = \lim_{x\to-3} \frac{x-3}{(x-3)(x+3)} = \lim_{x\to-3} \frac{1}{x+3}$$

Deze limiet bestaat niet, want $\lim_{x\uparrow-3} \frac{1}{x+3} = -\infty$ en $\lim_{x\downarrow-3} \frac{1}{x+3} = +\infty$

d. $$\lim_{x\to-3} \frac{x+3}{x^2-9} = \lim_{x\to-3} \frac{x+3}{(x+3)(x-3)} = \lim_{x\to-3} \frac{1}{x-3} = \frac{1}{-3-3} = -\frac{1}{6}$$

e. $$\lim_{x\uparrow-3} \frac{x^2+9}{x^2-9} = \left(\frac{18}{0^+}\right) = +\infty$$

f. $$\lim_{x\downarrow3} \frac{x^2+9}{x^2-9} = \left(\frac{18}{0^+}\right) = +\infty$$

3.

a. $$\lim_{x\downarrow0} \frac{x^2+4x}{x^2-4x} = \lim_{x\downarrow0} \frac{x(x+4)}{x(x-4)} = \lim_{x\downarrow0} \frac{x+4}{x-4} = \frac{4}{-4} = -1$$

b. $$\lim_{x\uparrow0} \frac{x^2+4x}{x^2-4x} = \lim_{x\uparrow0} \frac{x(x+4)}{x(x-4)} = \lim_{x\uparrow0} \frac{x+4}{x-4} = \frac{4}{-4} = -1$$

c. $$\lim_{x\to0} \frac{x^2+4x}{x^2-4x} = \lim_{x\to0} \frac{x(x+4)}{x(x-4)} = \lim_{x\to0} \frac{x+4}{x-4} = \frac{4}{-4} = -1$$

d. $$\lim_{x\downarrow4} \frac{x^2+4x}{x^2-4x} = \lim_{x\downarrow4} \frac{x(x+4)}{x(x-4)} = \lim_{x\downarrow4} \frac{x+4}{x-4} = \frac{8}{0^+} = +\infty$$

e. $$\lim_{x\uparrow4} \frac{x^2+4x}{x^2-4x} = \lim_{x\uparrow4} \frac{x(x+4)}{x(x-4)} = \lim_{x\uparrow4} \frac{x+4}{x-4} = \frac{8}{0^-} = -\infty$$

f. $\lim_{x\to4} \frac{x^2+4x}{x^2-4x}$ bestaat niet, want $\lim_{x\downarrow4} \frac{x^2+4x}{x^2-4x} = +\infty$ en $\lim_{x\uparrow4} \frac{x^2+4x}{x^2-4x} = -\infty$

g. $$\lim_{x\uparrow 2}\frac{x^2+2x}{x^2-4}=\lim_{x\uparrow 2}\frac{x(x+2)}{(x-2)(x+2)}=\lim_{x\uparrow 2}\frac{x}{x-2}=\frac{2}{0^-}=-\infty$$

h. $$\lim_{x\downarrow 2}\frac{x^2-2x}{x^2-4}=\lim_{x\downarrow 22}\frac{x(x+2)}{(x-2)(x+2)}=\lim_{x\downarrow 2}\frac{x}{x-2}=\frac{2}{0^+}=+\infty$$

i. $$\lim_{x\uparrow 2}\frac{x^2+2x}{(x-2)^2}=\lim_{x\uparrow 2}\frac{4+4}{0^+}=+\infty$$

j. $$\lim_{x\downarrow 2}\frac{x^2+2x}{(x-2)^2}=\lim_{x\downarrow 2}\frac{4+4}{0^+}=+\infty$$

k. $$\lim_{x\uparrow 2}\frac{x^2-2x}{(x-2)^2}=\lim_{x\uparrow 2}\frac{x(x-2)}{(x-2)^2}=\lim_{x\uparrow 2}\frac{x}{x-2}=\frac{2}{0^-}=-\infty$$

l. $$\lim_{x\downarrow 2}\frac{x^2-2x}{(x-2)^2}=\lim_{x\downarrow 2}\frac{x(x-2)}{(x-2)^2}=\lim_{x\downarrow 2}\frac{x}{x-2}=\frac{2}{0^+}=+\infty$$

4.

a. $$\lim_{t\to\infty}\frac{4t^3-1}{3t^2+2}=\lim_{t\to\infty}\frac{4\frac{t^3}{t^2}-\frac{1}{t^2}}{3\frac{t^2}{t^2}+\frac{2}{t^2}}=\lim_{t\to\infty}\frac{4t-\frac{1}{t^2}}{3+\frac{2}{t^2}}=\frac{\infty-0}{3+0}=\infty$$

b. $$\lim_{t\to-\infty}\frac{4t^3+1}{3t^2-2}=\lim_{t\to-\infty}\frac{4\frac{t^3}{t^2}+\frac{1}{t^2}}{3\frac{t^2}{t^2}-\frac{2}{t^2}}=\lim_{t\to-\infty}\frac{4t+\frac{1}{t^2}}{3-\frac{2}{t^2}}=\frac{-\infty+0}{3-0}=-\infty$$

c. $$\lim_{t\to\infty}\frac{4t^2-1}{3t^2+2}=\lim_{t\to\infty}\frac{4\frac{t^2}{t^2}-\frac{1}{t^2}}{3\frac{t^2}{t^2}+\frac{2}{t^2}}=\lim_{t\to\infty}\frac{4-\frac{1}{t^2}}{3+\frac{2}{t^2}}=\frac{4-0}{3+0}=\frac{4}{3}$$

d. $$\lim_{t\to-\infty}\frac{4t^2+1}{3t^2-2}=\lim_{t\to-\infty}\frac{4\frac{t^2}{t^2}+\frac{1}{t^2}}{3\frac{t^2}{t^2}-\frac{2}{t^2}}=\lim_{t\to-\infty}\frac{4+\frac{1}{t^2}}{3-\frac{2}{t^2}}=\frac{4+0}{3-0}=\frac{4}{3}$$

e. $$\lim_{t\to\infty}\frac{4t^3-1}{3t^4+2}=\lim_{t\to\infty}\frac{4\frac{t^3}{t^4}-\frac{1}{t^4}}{3\frac{t^4}{t^4}+\frac{2}{t^4}}=\lim_{t\to\infty}\frac{\frac{4}{t}-\frac{1}{t^4}}{3+\frac{2}{t^4}}=\frac{0-0}{3+0}=\frac{0}{3}=0$$

f. $$\lim_{t\to-\infty}\frac{4t^3-1}{3t^4+2}=\lim_{t\to-\infty}\frac{4\frac{t^3}{t^4}-\frac{1}{t^4}}{3\frac{t^4}{t^4}+\frac{2}{t^4}}=\lim_{t\to-\infty}\frac{\frac{4}{t}-\frac{1}{t^4}}{3+\frac{2}{t^4}}=\frac{0-0}{3+0}=\frac{0}{3}=0$$

g.

$$\lim_{u\to\infty}\frac{1+2\sqrt{u}-4u^4}{2u^2+\sqrt{u}+5}=\lim_{u\to\infty}\frac{\frac{1}{u^2}+2\frac{\sqrt{u}}{u^2}-4\frac{u^4}{u^2}}{2\frac{u^2}{u^2}+\frac{\sqrt{u}}{u^2}+\frac{5}{u^2}}=\lim_{u\to\infty}\frac{\frac{1}{u^2}+2\frac{1}{u\sqrt{u}}-4u^2}{2+\frac{1}{u\sqrt{u}}+\frac{5}{u^2}}$$

$$=\lim_{u\to\infty}\frac{0+2\cdot 0-4\cdot\infty}{2+0+5\cdot 0}=\frac{-\infty}{2}=-\infty$$

h.

$$\lim_{u\to\infty}\frac{3+4u^2}{5-2u^2+u\sqrt{u}}=\lim_{u\to\infty}\frac{\frac{3}{u^2}+4\frac{u^2}{u^2}}{\frac{5}{u^2}-2\frac{u^2}{u^2}+\frac{u\sqrt{u}}{u^2}}$$

$$=\lim_{u\to\infty}\frac{\frac{3}{u^2}+4}{\frac{5}{u^2}-2+\frac{1}{\sqrt{u}}}=\frac{0+4}{0-2+0}=\frac{4}{-2}=-2$$

5.

a. $$\lim_{x\to 0}\frac{\sin(-3x)}{5x}=\lim_{x\to 0}\frac{-\sin(3x)}{5x}\lim_{x\to 0}-\frac{3}{5}\cdot\frac{\sin(3x)}{3x}=-\frac{3}{5}\cdot 1=-\frac{3}{5}$$

b. $$\lim_{x\uparrow 0}\frac{\tan\left(\frac{1}{2}x\right)}{2x}=\lim_{x\uparrow 0}\frac{\frac{1}{2}}{2}\cdot\frac{\tan\left(\frac{1}{2}x\right)}{\frac{1}{2}x}=\frac{1}{4}\cdot 1=\frac{1}{4}$$

c. $$\lim_{x\uparrow 0}\frac{x}{\cos(2x)}=\frac{0}{1}=0$$

d.

$$\lim_{x\to 0}\frac{x\sin(2x)}{\tan^2(3x)}=\lim_{x\to 0}\left(\frac{3x}{\tan(3x)}\cdot\frac{1}{3}\right)\left(\frac{\sin(2x)}{2x}\cdot\frac{2x}{1}\right)\left(\frac{3x}{\tan(3x)}\cdot\frac{1}{3x}\right)$$

$$=\lim_{x\to 0}\frac{3x}{\tan(3x)}\frac{\sin(2x)}{2x}\frac{3x}{\tan(3x)}\cdot\frac{1}{3}\cdot\frac{2x}{1}\cdot\frac{1}{3x}$$

$$=\lim_{x\to 0}\frac{3x}{\tan(3x)}\frac{\sin(2x)}{2x}\frac{3x}{\tan(3x)}\cdot\frac{2}{9}=1\cdot 1\cdot 1\cdot\frac{2}{9}=\frac{2}{9}$$

e. $$\lim_{x\to 0}\frac{3x^2}{\sin x\cos x}=\lim_{x\to 0}\frac{x}{\sin x}\frac{3x}{\cos x}=1\cdot\frac{0}{1}=0$$

f.

$$\lim_{x\downarrow 0}\frac{5x^2}{(\sin(3x))^2\tan(2x)}=\lim_{x\downarrow 0}5\cdot\left(\frac{3x}{\sin(3x)}\cdot\frac{1}{3}\right)\left(\frac{3x}{\sin(3x)}\cdot\frac{1}{3}\right)\left(\frac{2x}{\tan(2x)}\cdot\frac{1}{2x}\right)$$

$$=\lim_{x\downarrow 0}\left(\frac{3x}{\sin(3x)}\right)\left(\frac{3x}{\sin(3x)}\right)\left(\frac{2x}{\tan(2x)}\right)\cdot 5\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{2x}$$

$$=1\cdot 1\cdot 1\cdot 5\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\infty=\infty$$

6.

a. $\lim\limits_{x\to\infty}\dfrac{\tan(3x)}{2x}$ bestaat niet, want $\tan(3x)$ gaat voor groter wordende waarden van x afwisselend naar $+\infty$ en $-\infty$.

b. Omdat $-1 \le \sin(3x) \le 1$ geldt $\dfrac{-1}{x^2} \le \dfrac{\sin(3x)}{x^2} \le \dfrac{1}{x^2}$. Ook geldt: $\lim\limits_{x\to\infty}\dfrac{1}{x^2} = 0$.

Vanwege de insluitstelling geldt dan: $\lim\limits_{x\to\infty}\dfrac{\sin(3x)}{x^2} = 0$

c. $\lim\limits_{x\to\infty}\dfrac{x}{\sin x}$ bestaat niet. Voor groter wordende waarden van x neemt $\sin x$ afwisselend positieve en negatieve waarden aan, waardoor $\dfrac{x}{\sin x}$ steeds sterker positieve en negatieve waarden aanneemt. Er treden ook steeds verticale asymptoten op.

d. $\lim\limits_{x\to-\infty}\dfrac{x^2}{(\sin x)^3}$ bestaat niet. Voor steeds sterker negatieve waarden van x neemt $(\sin x)^3$ afwisselend positieve en negatieve waarden aan, waardoor $\dfrac{x^2}{(\sin x)^3}$ steeds sterker positieve en negatieve waarden aanneemt. Er treden ook steeds verticale asymptoten op.

e $\lim\limits_{x\to-\infty}\dfrac{2^x}{\sin x}$ bestaat niet. Voor groter wordende waarden van x neemt $\sin x$ afwisselend positieve en negatieve waarden aan en 2^x wordt steeds sterker positief, waardoor $\dfrac{2^x}{\sin x}$ steeds sterker positieve en negatieve waarden aanneemt.
Er treden ook steeds verticale asymptoten op.

f Omdat $-1 \le \sin(3x) \le 1$ geldt $\dfrac{-1}{e^{2x}} \le \dfrac{\sin(3x)}{e^{2x}} \le \dfrac{1}{e^{2x}}$.

Ook geldt: $\lim\limits_{x\to\infty}\dfrac{1}{e^{2x}} = \lim\limits_{x\to\infty} e^{-2x} = 0$.

Vanwege de insluitstelling geldt dan: $\lim\limits_{x\to\infty}\dfrac{\sin(3x)}{e^{2x}} = 0$

7.

a.. V.A.: $t = -2$; H.A.: $y = 0$
b. V.A.: geen; H.A.: $y = 3$
c. V.A.: $x = 0$ $x = -\frac{1}{2}$; H.A.: $y = 0$
d. V.A.: $x = 0$, $x = -\frac{1}{2}$ en $x = \frac{1}{2}$; H.A.: $y = 0$
e. V.A.: $u = 0$; H.A.: $y = 2$
f. V.A.: $u = 0$; H.A.: $y = 0$

8.

a. $f(x) = \dfrac{x^2 + 5x + 6}{x + 5}$; oneindige discontinuïteit voor $x = -5$.

b. $f(x) = \dfrac{x^2 + 5x + 6}{x + 3}$; ophefbare discontinuïteit voor $x = -3$;.

c. $f(x) = \dfrac{x + 2}{x^2 + x - 2}$; ophefbare discontinuïteit voor $x = -2$ en oneindige discontinuïteit voor $x = 1$.

d. $f(x) = \dfrac{x^2 - 1}{|x - 1|}$; eindige sprong discontinuïteit voor $x = 1$.

9.

a. $$f(x) = \begin{cases} \dfrac{x^2 - 4}{2x - 4} & \text{voor} \quad x \neq 2 \\ 2 & \text{voor} \quad x = 2 \end{cases}$$

Deze functie is nergens discontinu

b.

$$f(x) = \begin{cases} x^2 - 2 & \text{voor} \quad x \leq -1 \\ \dfrac{1}{\tan\left(\frac{1}{4}\pi x\right)} & \text{voor} \quad -1 < x < 1 \\ x^2 - 2 & \text{voor} \quad x \geq 1 \end{cases}$$

Eindige sprongdiscontinuïteit voor $x = 1$
(N.B. Voor $x = -1$ is de functie wel continu)

10.

a.

$$f(x) = \begin{cases} \dfrac{\sin(3x)}{5x} & \text{voor} \quad x \neq 0 \\ a & \text{voor} \quad x = 0 \end{cases}$$

Omdat $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \dfrac{\sin(3x)}{5x} = \dfrac{3}{5}$ moet voor *a* de waarde $\dfrac{3}{5}$ gekozen worden.

b.

$$f(x) = \begin{cases} \dfrac{\cos(3x)}{5x} & \text{voor} \quad x \neq 0 \\ a & \text{voor} \quad x = 0 \end{cases}$$

$\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \dfrac{\cos(3x)}{5x}$ bestaat niet, omdat $\lim\limits_{x \uparrow 0} f(x) = \lim\limits_{x \to 0} \dfrac{\cos(3x)}{5x} = \dfrac{1}{0^-} = -\infty$ en $\lim\limits_{x \downarrow 0} f(x) = \lim\limits_{x \downarrow 0} \dfrac{\cos(3x)}{5x} = \dfrac{1}{0^+} = +\infty$. Er is daarom geen enkele waarde van *a* waarvoor de functie *f* continu is.

c.

$$f(x)=\begin{cases}\dfrac{x^3-x}{|x|} & \text{voor} \quad x\neq 0\\ a & \text{voor} \quad x=0\end{cases}$$

$\lim\limits_{x\to 0} f(x)$ bestaat niet, omdat linker- en rechterlimiet ongelijk zijn aan elkaar:

$$\lim_{x\uparrow 0} f(x)=\lim_{x\uparrow 0}\frac{x^3-x}{|x|}=\lim_{x\uparrow 0}\frac{x\left(x^2-1\right)}{-x}=\lim_{x\uparrow 0}\frac{\left(x^2-1\right)}{-1}=1 \text{ en}$$

$$\lim_{x\downarrow 0} f(x)=\lim_{x\downarrow 0}\frac{x^3-x}{|x|}=\lim_{x\downarrow 0}\frac{x\left(x^2-1\right)}{x}=\lim_{x\downarrow 0}\left(x^2-1\right)=-1.$$

Er is daarom geen enkele waarde van *a* waarvoor de functie *f* continu is.

11.

a. $\lim\limits_{x\to\infty}\arccos(1-e^{-2x})=\arccos(1-0)=0$

b. $\lim\limits_{x\downarrow 0}\dfrac{e^{\tan 3x}}{5x}=\left(\dfrac{1}{0^+}\right)=\infty$

c. $\lim\limits_{x\downarrow 0}\dfrac{\tan 3x}{5x}=\lim\limits_{x\downarrow 0}\dfrac{\tan 3x}{3x}\cdot\dfrac{3}{5}=1\cdot\dfrac{3}{5}=\dfrac{3}{5}$ en dus $\lim\limits_{x\downarrow 0} e^{\frac{\tan 3x}{5x}}=e^{\frac{3}{5}}\approx 1{,}822$

12.

a. $\lim\limits_{t\downarrow 0}\dfrac{\tan 3t}{2t}=\lim\limits_{t\downarrow 0}\dfrac{\tan 3t}{3t}\cdot\dfrac{3}{2}=\dfrac{3}{2}$ en dus $\lim\limits_{t\downarrow 0}\ln\left(\dfrac{\tan 3t}{2t}\right)=\ln\left(\dfrac{3}{2}\right)\approx 0{,}405$

b. Omdat $-1\leq\cos(3t)\leq 1$ en $t\geq 0$ (we nemen een limiet voor $t\to\infty$) geldt:

$\dfrac{-1}{2t}\leq\dfrac{\cos 3t}{2t}\leq\dfrac{1}{2t}$. Ook geldt: $\lim\limits_{t\to\infty}\dfrac{1}{2t}=0$.

Vanwege de insluitstelling geldt dan: $\lim\limits_{t\to\infty}\dfrac{\cos(3t)}{2t}=0$. Ook geldt: $\left|\dfrac{\cos 3t}{2t}\right|\geq 0$

Daarom is $\lim\limits_{t\to\infty}\ln\left|\dfrac{\cos 3t}{2t}\right|=\ln\left(\lim\limits_{t\to\infty}\left|\dfrac{\cos 3t}{2t}\right|\right)=\ln\left(0^+\right)=-\infty$

13.

a. $\dfrac{\Delta y}{\Delta x}=\dfrac{f(1)-f(0)}{1-0}=\dfrac{0-0}{1-0}=0.$

b. $\dfrac{\Delta y}{\Delta x}=\dfrac{f(0{,}1)-f(0)}{0{,}1-0}=\dfrac{(0{,}01-0{,}1)-0}{0{,}1-0}=\dfrac{-0{,}09}{0{,}1}=-0{,}9$

c. $\dfrac{\Delta y}{\Delta x}=\dfrac{f(3{,}9)-f(4)}{3{,}9-4}=\dfrac{13{,}24-14}{3{,}9-4}=\dfrac{-0{,}76}{-0{,}1}=7{,}6.$

d. $\dfrac{\Delta y}{\Delta x}=\dfrac{f(0{,}9)-f(1)}{0{,}9-1}=\dfrac{-0{,}139-0}{0{,}9-1}=\dfrac{-0{,}139}{-0{,}1}=1{,}39.$

14.

a. $\frac{\Delta y}{\Delta x}=\frac{f(1)-f(0)}{1-0}=\frac{1-3}{1-0}=-2$.

b. $\frac{\Delta y}{\Delta x}=\frac{f(-2)-f(-3)}{(-2)-(-3)}=\frac{7-9}{(-2)-(-3)}=-2$

c. $\frac{\Delta y}{\Delta x}=\frac{f(a-1)-f(a)}{(a-1)-a}=\frac{3-2(a-1)-(3-2a)}{(a-1)-a}=\frac{3-2a+2-3+2a}{-1}=-2$

d.

$$\frac{\Delta y}{\Delta x}=\frac{f(a+\Delta a)-f(a)}{(a+\Delta a)-a}=\frac{3-2(a+\Delta a)-(3-2a)}{(a+\Delta a)-a}=$$

$$=\frac{3-2a-2\Delta a-3+2a}{\Delta a}=\frac{-2\Delta a}{\Delta a}=-2$$

15.

a.

$$\frac{\Delta y}{\Delta t}=\frac{f(t+\Delta t)-f(t)}{\Delta t}=\frac{\left(2(t+\Delta t)^2-3(t+\Delta t)\right)-\left(2t^2-3t\right)}{\Delta t}=$$

$$=\frac{\left(2t^2+4t\Delta t+2(\Delta t)^2-3t-3\Delta t\right)-\left(2t^2-3t\right)}{\Delta t}=$$

$$=\frac{4t\Delta t+2(\Delta t)^2-3\Delta t}{\Delta t}=4t+2\Delta t-3=4t-3+2\Delta t$$

b.

$$\frac{\Delta z}{\Delta u}=\frac{f(u+\Delta u)-f(u)}{\Delta u}=\frac{\left((u+\Delta u)+1\right)^2-(u+1)^2}{\Delta u}=$$

$$=\frac{\left(u^2+(\Delta u)^2+1^2+2u\Delta u+2u\cdot 1+2\Delta u\cdot 1\right)-\left(u^2+2u+1\right)}{\Delta u}=$$

$$=\frac{(\Delta u)^2+2u\Delta u+2\Delta u}{\Delta u}=\Delta u+2u+2=2u+2+\Delta u$$

16.

a

$$f'(1)=\lim_{\Delta t\to 0}\frac{f(1+\Delta t)-f(1)}{\Delta t}=\lim_{\Delta t\to 0}\frac{\left((1+\Delta t)^2-3(1+\Delta t)\right)-\left(1^2-3\cdot 1\right)}{\Delta t}=$$

$$=\lim_{\Delta t\to 0}\frac{\left(1+2\Delta t+(\Delta t)^2-3-3\Delta t\right)-(-2)}{\Delta t}=\lim_{\Delta t\to 0}\frac{-\Delta t+(\Delta t)^2}{\Delta t}=$$

$$=\lim_{\Delta t\to 0}(-1+\Delta t)=-1$$

b

$$f'(t) = \lim_{\Delta t\to 0}\frac{f(t+\Delta t)-f(t)}{\Delta t} = \lim_{\Delta t\to 0}\frac{\left((t+\Delta t)^2-3(t+\Delta t)\right)-\left(t^2-3t\right)}{\Delta t} =$$
$$= \lim_{\Delta t\to 0}\frac{\left(t^2+2t\Delta t+(\Delta t)^2-3t-3\Delta t\right)-\left(t^2-3t\right)}{\Delta t}$$
$$= \lim_{\Delta t\to 0}\frac{2t\Delta t-3\Delta t+(\Delta t)^2}{\Delta t} = \lim_{\Delta t\to 0}\left(2t-3+\Delta t\right) = 2t-3$$

17.

a. $f(x) = -2x^4+3x^2-4 \Rightarrow f'(x) = -2\cdot 4x^3+3\cdot 2x^1-0 = -8x^3+6x$

b. $f(x) = x\sqrt[4]{x^3}-\frac{8}{3}\sqrt{x}+3x-1 = x^{\frac{7}{4}}-\frac{8}{3}x^{\frac{1}{2}}+3x-1 \Rightarrow$

$f'(x) = \frac{7}{4}x^{\frac{3}{4}}-\frac{8}{3}\cdot\frac{1}{2}x^{-\frac{1}{2}}+3-0 = \frac{7}{4}\sqrt[4]{x^3}-\frac{4}{3\sqrt{x}}+3$

c. $f(x) = \frac{4}{x^2\sqrt{x}}-\frac{1}{\sqrt{x}}+x^3\sqrt{x}+\sqrt{5} = 4x^{-\frac{5}{2}}-x^{-\frac{1}{2}}+x^{\frac{7}{2}}+\sqrt{5} \Rightarrow$

$f'(x) = 4\cdot\left(-\frac{5}{2}\right)x^{-\frac{7}{2}}-\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}+\frac{7}{2}x^{\frac{5}{2}}+0 = -\frac{10}{x^3\sqrt{x}}+\frac{1}{2x\sqrt{x}}+\frac{7}{2}x^2\sqrt{x}$

d. $f(x) = \frac{1}{x}+\frac{1}{\pi^3} = x^{-1}+\frac{1}{\pi^3} \Rightarrow f(x) = -1\cdot x^{-2}+0 = -\frac{1}{x^2}$

18.

a. $f(x) = -4\tan x+\sin x+2x \Rightarrow f(x) = -\frac{4}{\cos^2 x}+\cos x+2$

b. $g(x) = \tan x-\frac{1}{\cos x} \Rightarrow$

$g'(x) = \frac{1}{\cos^2 x}-\frac{\cos x\cdot 0-1\cdot(-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x}-\frac{\sin x}{\cos^2 x} = \frac{1-\sin x}{\cos^2 x}$

c. $f(t) = \sin t\tan t \Rightarrow f'(t) = \cos t\tan t+\sin t\cdot\frac{1}{\cos^2 t} = \sin t+\frac{\sin t}{\cos^2 t}$

d. $h(t) = x\sin t+x^2\sqrt{t} \Rightarrow h(t) = x\cos t+x^2\cdot\frac{1}{2\sqrt{t}} = x\cos t+\frac{x^2}{2\sqrt{t}}$

19.

a. $f(x) = \frac{3}{\sqrt{x}}+\frac{x}{\sqrt{3}} = 3x^{-\frac{1}{2}}+\frac{1}{\sqrt{3}}x \Rightarrow$

$f'(x) = 3\cdot\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}+\frac{1}{\sqrt{3}} = -\frac{3}{2x\sqrt{x}}+\frac{1}{3}\sqrt{3}$

b. $g(y) = \frac{3y^4}{4}+\frac{2}{5y^5} = \frac{3}{4}y^4+\frac{2}{5}y^{-5} \Rightarrow$

$g(y) = \frac{3}{4}\cdot 4y^3+\frac{2}{5}\cdot(-5)y^{-6} = 3y^3-\frac{2}{y^6}$

c. $h(t) = \left(3t^3-2t+5\right)(4-7t) \Rightarrow$

$$h'(t) = (9t^2 - 2)(4 - 7t) + (3t^3 - 2t + 5)\cdot(-7)$$
$$= (9t^2 - 2)(4 - 7t) - 7(3t^3 - 2t + 5)$$

d. $k(t) = \dfrac{t^2 - 1}{t^2 + 1} \Rightarrow$

$$k'(t) = \frac{(t^2+1)\cdot(2t) - (t^2-1)\cdot(2t)}{(t^2+1)^2} = \frac{2t^3 + 2t - 2t^3 + 2t}{(t^2+1)^2} = \frac{4t}{(t^2+1)^2}$$

20.

a. $h(t) = \dfrac{t^2}{\tan t} \Rightarrow$

$$h'(t) = \frac{(\tan t)\cdot 2t - t^2 \cdot \dfrac{1}{\cos^2 t}}{(\tan t)^2} = \frac{2t\cdot\tan t\cdot\cos^2 t - t^2}{\cos^2 t\cdot\tan^2 t} = \frac{2t\cdot\sin t\cdot\cos t - t^2}{\sin^2 t}$$

b.

$$g(t) = \frac{t + \tan t}{\cos t} \Rightarrow$$

$$g'(t) = \frac{(\cos t)\left(1 + \dfrac{1}{\cos^2 t}\right) - (t + \tan t)(-\sin t)}{(\cos t)^2}$$

$$= \frac{\cos t + \dfrac{1}{\cos t} + t\sin t + \tan t\cdot\sin t}{\cos^2 t}$$

$$= \frac{\cos^2 t + 1 + t\sin t\cdot\cos t + \cos t\tan t\sin t}{\cos^3 t}$$

$$= \frac{\cos^2 t + 1 + t\sin t\cdot\cos t + \sin^2 t}{\cos^3 t} = \frac{2 + t\sin t\cdot\cos t}{\cos^3 t}$$

c. $f(x) = \sqrt[3]{x}\sqrt{x} - x^2\sqrt{3} = x^{\frac{1}{3}}x^{\frac{1}{2}} - x^2\sqrt{3} = x^{\frac{5}{6}} - \sqrt{3}\cdot x^2 \Rightarrow$

$$f'(x) = \tfrac{5}{6}x^{-\frac{1}{6}} - \sqrt{3}\cdot 2x = \frac{5}{6\sqrt[6]{x}} - 2\sqrt{3}\cdot x$$

d. $h(x) = x^2 \sin x \tan x \Rightarrow$

$$h'(x) = 2x\sin x\tan x + x^2\cos x\tan x + x^2\sin x\cdot\frac{1}{\cos^2 x}$$
$$= 2x\sin x\tan x + x^2\sin x + \frac{x^2\sin x}{\cos^2 x}$$

21.

Uit $f(x) = \frac{1}{3}x^2 + 2x - 1$ volgt: $f'(x) = \frac{2}{3}x + 2$.

a. Het bedoelde punt is $(1, f(1)) = \left(1, \frac{4}{3}\right)$.

De richtingscoëfficiënt van de raaklijn in dit punt is $f'(1) = 2\frac{2}{3} = \frac{8}{3}$.

De vergelijking van de raaklijn is dan $y = \frac{8}{3}x + b$. Deze lijn gaat door het punt $\left(1, \frac{4}{3}\right)$.

Invullen van dit punt geeft: $\frac{4}{3} = \frac{8}{3} \cdot 1 + b \Rightarrow b = -\frac{4}{3}$.

De vergelijking van de gevraagde raaklijn is daarmee $y = \frac{8}{3}x - \frac{4}{3}$.

b. Het bedoelde punt is $(0, f(0)) = (0, -1)$.

De richtingscoëfficiënt van de raaklijn in dit punt is $f'(0) = 2$.

De vergelijking van de raaklijn is dan $y = 2x + b$. Deze lijn gaat door het punt $(0, -1)$.

Invullen van dit punt geeft: $-1 = 2 \cdot 0 + b \Rightarrow b = -1$.

De vergelijking van de gevraagde raaklijn is daarmee $y = 2x - 1$.

c. Een horizontale raaklijn heeft richtingscoëfficiënt 0, dus voor het bedoelde punt geldt:

$f'(x) = 0$ en dus $\frac{2}{3}x + 2 = 0 \Rightarrow x = -3$

Het bedoelde punt is dus $(-3, f(-3)) = (-3, -4)$

De vergelijking van de raaklijn is dan $y = 0x + b$, dus $y = b$. Deze lijn gaat door het punt $(-3, -4)$. Invullen van dit punt geeft: $-4 = b \Rightarrow b = -4$.

De vergelijking van de gevraagde horizontale raaklijn is dus $y = -4$.

22.

a. $$f(x) = \left(2x^2 - 3x + 5\right)^7 \Rightarrow f''(x) = 7 \cdot \left(2x^2 - 3x + 5\right)^6 \cdot (4x - 3)$$

b. $$f(t) = \left(t^4 + 3t^2 - 1\right)^{-2} \Rightarrow$$
$$f(t) = -2 \cdot \left(t^4 + 3t^2 - 1\right)^{-3} \cdot \left(4t^3 + 6t\right) = -4t\left(2t^2 + 3\right)\left(t^4 + 3t^2 - 1\right)^{-3}$$

c. $$g(t) = \left(t^3 + 2\left(t^2 - 4\right)^{-3}\right)^5 \Rightarrow$$
$$g'(t) = 5\left(t^3 + 2\left(t^2 - 4\right)^{-3}\right)^4 \cdot \left(3t^2 + 2 \cdot (-3)\left(t^2 - 4\right)^{-4} \cdot 2t\right)$$
$$= 5\left(t^3 + 2\left(t^2 - 4\right)^{-3}\right)^4 \cdot \left(3t^2 - 12t\left(t^2 - 4\right)^{-4}\right)$$

d. $$f(x) = \left(x^5 - 3x\right)^5 \cdot \left(1 + 3x^3\right)^3 \Rightarrow$$
$$f'(x) = 5\left(x^5 - 3x\right)^4\left(5x^4 - 3\right)\left(1 + 3x^3\right)^3 + \left(x^5 - 3x\right)^5 \cdot 3\left(1 + 3x^3\right)^2 \cdot 9x^2$$
$$= \left(x^5 - 3x\right)^4\left(1 + 3x^3\right)^2\left(5\left(5x^4 - 3\right)\left(1 + 3x^3\right) + \left(x^5 - 3x\right) \cdot 3 \cdot 9x^2\right)$$
$$= \left(x^5 - 3x\right)^4\left(1 + 3x^3\right)^2\left(25x^4 + 75x^7 - 15 - 45x^3 + 27x^7 - 81x^3\right)$$
$$= \left(x^5 - 3x\right)^4\left(1 + 3x^3\right)^2\left(102x^7 + 25x^4 - 126x^3 - 15\right)$$

23.

a.
$$h(u)=\frac{3u^2+2u+1}{5u-4}\Rightarrow$$
$$h'(u)=\frac{(5u-4)(6u+2)-\left(3u^2+2u+1\right)\cdot 5}{(5u-4)^2}$$
$$=\frac{\left(30u^2+10u-24u-8\right)-\left(15u^2+10u+5\right)}{(5u-4)^2}=\frac{15u^2-24u-13}{(5u-4)^2}$$

b.
$$k(t)=\frac{t^2+1}{\left(t^2-1\right)^2}\Rightarrow$$
$$k'(t)=\frac{\left(t^2-1\right)^2\cdot 2t-\left(t^2+1\right)\cdot 2\left(t^2-1\right)\cdot 2t}{\left(t^2-1\right)^4}$$
$$=\frac{2t\left(t^2-1\right)\left(\left(t^2-1\right)-2\left(t^2+1\right)\right)}{\left(t^2-1\right)^4}=\frac{2t\left(-t^2-3\right)}{\left(t^2-1\right)^3}=\frac{-2t\left(t^2+3\right)}{\left(t^2-1\right)^3}$$

of

$$k(t)=\frac{t^2+1}{\left(t^2-1\right)^2}=\left(t^2+1\right)\left(t^2-1\right)^{-2}\Rightarrow$$
$$k'(t)=2t\left(t^2-1\right)^{-2}+\left(t^2+1\right)(-2)\left(t^2-1\right)^{-3}\cdot 2t$$
$$=\frac{2t}{\left(t^2-1\right)^2}-\frac{4t\left(t^2+1\right)}{\left(t^2-1\right)^3}=\frac{2t\left(t^2-1\right)-4t\left(t^2+1\right)}{\left(t^2-1\right)^3}=$$
$$=\frac{2t\left(t^2-1-2t^2-2\right)}{\left(t^2-1\right)^3}=\frac{2t\left(-t^2-3\right)}{\left(t^2-1\right)^3}=\frac{-2t\left(t^2+3\right)}{\left(t^2-1\right)^3}$$

c.
$$g(t)=\left(\frac{t^2+6^3}{5-3t}\right)^{-5}\Rightarrow$$
$$g'(t)=-5\cdot\left(\frac{t^2+6^3}{5-3t}\right)^{-6}\cdot\frac{(5-3t)(2t+0)-\left(t^2+6^3\right)\cdot(-3)}{(5-3t)^2}$$
$$=-5\cdot\left(\frac{5-3t}{t^2+6^3}\right)^{6}\cdot\frac{10t-6t^2+3t^2+3\cdot 6^3}{(5-3t)^2}$$
$$=\frac{-5\cdot(5-3t)^4\left(-3t^2+10t+648\right)}{\left(t^2+216\right)^6}=\frac{5(5-3t)^4\left(3t^2-10t-648\right)}{\left(t^2+216\right)^6}$$

of

$$g(t)=\left(\frac{t^2+6^3}{5-3t}\right)^{-5}=\left(\frac{5-3t}{t^2+6^3}\right)^{5}\Rightarrow$$

$$g'(t) = 5\cdot\left(\frac{5-3t}{t^2+6^3}\right)^4\cdot\frac{\left(t^2+6^3\right)\cdot(-3)-(5-3t)(2t+0)}{\left(t^2+6^3\right)^2}$$

$$= 5\cdot\left(\frac{5-3t}{t^2+6^3}\right)^4\cdot\frac{-3t^2-3\cdot 6^3-10t+6t^2}{\left(t^2+6^3\right)^2} = \frac{5\cdot(5-3t)^4\left(3t^2-10t-648\right)}{\left(t^2+216\right)^6}$$

d. $f(x) = \dfrac{\left(x^2+1\right)^3}{(5-3x)^4} \Rightarrow$

$$f'(x) = \frac{(5-3x)^4\cdot 3\left(x^2+1\right)^2\cdot 2x-\left(x^2+1\right)^3\cdot 4\cdot(5-3x)^3\cdot(-3)}{(5-3x)^8}$$

$$= \frac{3\cdot 2\cdot(5-3x)^3\left(x^2+1\right)^2\left((5-3x)\cdot x-\left(x^2+1\right)\cdot 2\cdot(-1)\right)}{(5-3x)^8}$$

$$= \frac{6\left(x^2+1\right)^2\left(5x-3x^2+2x^2+2\right)}{(5-3x)^5} = \frac{-6\left(x^2+1\right)^2\left(x^2-5x-2\right)}{(5-3x)^5}$$

of

$$f(x) = \frac{\left(x^2+1\right)^3}{(5-3x)^4} = \left(x^2+1\right)^3\cdot(5-3x)^{-4} \Rightarrow$$

$$f'(x) = 3\left(x^2+1\right)^2\cdot 2x\cdot(5-3x)^{-4}+\left(x^2+1\right)^3\cdot(-4)\cdot(5-3x)^{-5}\cdot(-3)$$

$$= \frac{6x\left(x^2+1\right)^2}{(5-3x)^4}+\frac{12\left(x^2+1\right)^3}{(5-3x)^5} = \frac{6x\left(x^2+1\right)^2(5-3x)+12\left(x^2+1\right)^3}{(5-3x)^5}$$

$$= \frac{6\left(x^2+1\right)^2\cdot\left(x(5-3x)+2\left(x^2+1\right)\right)}{(5-3x)^5} = \frac{6\left(x^2+1\right)^2\cdot\left(-x^2+5x+2\right)}{(5-3x)^5}$$

$$= \frac{-6\left(x^2+1\right)^2\left(x^2-5x-2\right)}{(5-3x)^5}$$

24.

a. $f(x) = \dfrac{1}{\sqrt[5]{x^4+2}} = \left(x^4+2\right)^{-\frac{1}{5}} \Rightarrow$

$$f'(x) = -\tfrac{1}{5}\left(x^4+2\right)^{-1\frac{1}{5}}\cdot 4x^3 = -\frac{4x^3}{5\left(x^4+2\right)\sqrt[5]{x^4+2}}$$

b. $f(x) = \dfrac{1}{\sqrt{3x-5}} = (3x-5)^{-\frac{1}{2}} \Rightarrow f(x) = -\tfrac{1}{2}(3x-5)^{-1\frac{1}{2}}\cdot 3 = -\dfrac{3}{2(3x-5)\sqrt{3x-5}}$

c. $g(t) = \dfrac{t}{\sqrt{6t^4+5}} = t\cdot\left(6t^4+5\right)^{-\frac{1}{2}} \Rightarrow$

$$g(t) = \left(6t^4+5\right)^{-\frac{1}{2}} + t \cdot -\tfrac{1}{2}\left(6t^4+5\right)^{-1\frac{1}{2}} \cdot 24t^3 = \frac{1}{\sqrt{6t^4+5}} - \frac{12t^4}{\left(6t^4+5\right)\sqrt{6t^4+5}}$$

$$= \frac{6t^4+5}{\left(6t^4+5\right)\sqrt{6t^4+5}} - \frac{12t^4}{\left(6t^4+5\right)\sqrt{6t^4+5}} = \frac{-6t^4+5}{\left(6t^4+5\right)\sqrt{6t^4+5}}$$

d. $$g(t) = \sqrt{2t-\sqrt{3t}} = \left(2t-\sqrt{3}\cdot t^{\frac{1}{2}}\right)^{\frac{1}{2}} \Rightarrow$$

$$g(t) = \frac{1}{2}\left(2t-\sqrt{3}\cdot t^{\frac{1}{2}}\right)^{-\frac{1}{2}} \cdot \left(2-\sqrt{3}\cdot\tfrac{1}{2}t^{-\frac{1}{2}}\right) = \frac{2-\dfrac{\sqrt{3}}{2\sqrt{t}}}{2\sqrt{2t-\sqrt{3t}}} = \frac{4\sqrt{t}-\sqrt{3}}{4\sqrt{t}\sqrt{2t-\sqrt{3t}}}$$

25.

a. $$f(x) = \frac{\tan^4 x}{\cos^3 x} = \frac{(\tan x)^4}{(\cos x)^3} \Rightarrow$$

$$f'(x) = \frac{(\cos x)^3 \cdot 4(\tan x)^3 \cdot \dfrac{1}{(\cos x)^2} - (\tan x)^4 \cdot 3(\cos x)^2(-\sin x)}{(\cos x)^6}$$

$$= \frac{4(\cos x)(\tan x)^3 + 3(\tan x)^4(\cos x)^2(\sin x)}{(\cos x)^6}$$

$$= \frac{4(\tan x)^3 + 3(\tan x)^4(\cos x)(\sin x)}{(\cos x)^5}$$

$$= \frac{4\tan^3 x + 3\tan^4 x \cdot \cos x \cdot \sin x}{(\cos x)^5} = \frac{4\dfrac{\sin^3 x}{\cos^3 x} + 3\dfrac{\sin^4 x}{\cos^4 x}\cdot \cos x \cdot \sin x}{(\cos x)^5}$$

$$= \frac{4\dfrac{\sin^3 x}{\cos^3 x} + 3\dfrac{\sin^5 x}{\cos^3 x}}{(\cos x)^5} = \frac{4\sin^3 x + 3\sin^5 x}{(\cos x)^8}$$

b. $$f(x) = \sqrt{\frac{\sin^2 x}{\cos^3 x}} = \left(\frac{\sin^2 x}{\cos^3 x}\right)^{\frac{1}{2}} \Rightarrow$$

$$f'(x) = \frac{1}{2}\left(\frac{\sin^2 x}{\cos^3 x}\right)^{-\frac{1}{2}} \cdot \frac{\cos^3 x \cdot 2\sin x \cdot \cos x - \sin^2 x \cdot 3\cos^2 x \cdot (-\sin x)}{\cos^6 x}$$

$$= \left(\frac{\cos^3 x}{\sin^2 x}\right)^{\frac{1}{2}} \cdot \frac{2\cos^2 x \cdot \sin x + 3\sin^3 x}{2\cos^4 x}$$

$$= \sqrt{\frac{\cos^3 x}{\sin^2 x}} \cdot \frac{2\cos^2 x \cdot \sin x + 3\sin^3 x}{2\cos^4 x}$$

c. $$f(t) = \sin^3(2t) + \cos^3(2t) \Rightarrow$$

$$f'(t) = 3\sin^2(2t)\cdot\cos(2t)\cdot 2 + 3\cos^2(2t)\cdot(-\sin(2t))\cdot 2$$
$$= 6\sin(2t)\cos(2t)(\sin(2t)-\cos(2t)) = 3\sin(4t)(\sin(2t)-\cos(2t))$$

d. $$f(t) = \tan^4(2t) + \frac{1}{\cos(2t)} = \tan^4(2t) + (\cos(2t))^{-1} \Rightarrow$$
$$f'(t) = 4\tan^3(2t)\cdot\frac{1}{\cos^2(2t)}\cdot 2 + (-1)(\cos(2t))^{-2}\cdot(-\sin(2t))\cdot 2$$
$$= \frac{8\tan^3(2t)}{\cos^2(2t)} + \frac{2\sin(2t)}{\cos^2(2t)} = \frac{8\tan^3(2t)+2\sin(2t)}{\cos^2(2t)}$$

26.

a. $$f(x) = e^{-x^3} \Rightarrow f'(x) = e^{-x^3}\cdot(-3x^2) = -3x^2e^{-x^3}$$

b. $$f(x) = xe^{\sin(2x)} \Rightarrow$$
$$f'(x) = 1\cdot e^{\sin(2x)} + xe^{\sin(2x)}\cdot\cos(2x)\cdot 2 = (1+2x\cos(2x))\cdot e^{\sin(2x)}$$

c. $$f(x) = \frac{e^{x+\sin(2x)}}{\tan(3x)} \Rightarrow$$
$$f'(x) = \frac{\tan(3x)\cdot e^{x+\sin(2x)}\cdot(1+\cos(2x)\cdot 2) - e^{x+\sin(2x)}\cdot\frac{1}{\cos^2(3x)}\cdot 3}{(\tan(3x))^2}$$
$$= \frac{\cos^2(3x)\cdot\tan(3x)\cdot e^{x+\sin(2x)}\cdot(1+\cos(2x)\cdot 2) - 3e^{x+\sin(2x)}}{\cos^2(3x)\cdot\tan^2(3x)}$$
$$= \frac{(\cos(3x)\cdot\sin(3x)\cdot(1+2\cos(2x))-3)\cdot e^{x+\sin(2x)}}{\sin^2(3x)}$$
$$= \frac{(\cos(3x)\cdot\sin(3x)+2\cos(2x)\cos(3x)\sin(3x)-3)\cdot e^{x+\sin(2x)}}{\sin^2(3x)}$$

d. $$f(x) = e^{x^3\sin\left(2x-\frac{1}{3}\pi\right)} \Rightarrow$$
$$f'(x) = e^{x^3\sin\left(2x-\frac{1}{3}\pi\right)}\cdot\left(3x^2\sin\left(2x-\tfrac{1}{3}\pi\right) + x^3\cos\left(2x-\tfrac{1}{3}\pi\right)\cdot 2\right)$$
$$= x^2\left(3\sin\left(2x-\tfrac{1}{3}\pi\right)+2x\cos\left(2x-\tfrac{1}{3}\pi\right)\right)e^{x^3\sin\left(2x-\frac{1}{3}\pi\right)}$$

27.

a. $$g(t) = \ln(t^3) \Rightarrow g'(t) = \frac{1}{t^3}\cdot 3t^2 = \frac{3}{t}$$

of

$$g(t) = \ln(t^3) = 3\ln(t) \Rightarrow g'(t) = 3\cdot\frac{1}{t} = \frac{3}{t}$$

b. $$g(t) = (\ln t)^3 \Rightarrow g'(t) = 3(\ln t)^2\cdot\frac{1}{t} = \frac{3\ln^2 t}{t} = \frac{3}{t}\cdot\ln^2 t$$

c. $$g(t) = \ln(\ln(\ln t)) \Rightarrow g'(t) = \frac{1}{\ln(\ln t)}\cdot\frac{1}{\ln t}\cdot\frac{1}{t} = \frac{1}{t\cdot\ln t\cdot\ln(\ln t)}$$

d. $g(t) = \sqrt{\ln(\sin(3t))} \Rightarrow$

$$g'(t) = \frac{1}{2\sqrt{\ln(\sin(3t))}} \cdot \frac{1}{\sin(3t)} \cdot \cos(3t) \cdot 3$$

$$= \frac{3\cos(3t)}{2\sin(3t)\sqrt{\ln(\sin(3t))}} = \frac{3}{2\tan(3t)\sqrt{\ln(\sin(3t))}}$$

28.

a. $f(t) = 7^{2t^3-3t} \Rightarrow f'(t) = 7^{2t^3-3t} \cdot \ln 7 \cdot (6t-3) = 3\ln 7 \cdot (2t-1) \cdot 7^{2t^3-3t}$

b. $f(t) = 2^{{}^3\log(4t+5)} \Rightarrow$

$$f'(t) = 2^{{}^3\log(4t+5)} \cdot \ln 2 \cdot \frac{1}{(4t+5)\ln 3} \cdot 4 = \frac{4 \cdot \ln(2)}{\ln(3) \cdot (4t+5)} \cdot 2^{{}^3\log(4t+5)}$$

c. $g(x) = \ln\left(xe^x\right) \Rightarrow g'(x) = \frac{1}{xe^x}\left(1 \cdot e^x + x \cdot e^x\right) = \frac{e^x(1+x)}{xe^x} = \frac{1+x}{x} = \frac{1}{x} + 1$

of

$$g(x) = \ln\left(xe^x\right) = \ln(x) + \ln\left(e^x\right) = \ln(x) + x \Rightarrow g'(x) = \frac{1}{x} + 1$$

d. $h(t) = \dfrac{\ln(2t^3)}{t^6} \Rightarrow$

$$h'(t) = \frac{t^6 \cdot \left(\frac{1}{2t^3} \cdot 6t^2\right) - \ln(2t^3) \cdot 6t^5}{\left(t^6\right)^2} = \frac{3t^5 - \ln(2t^3) \cdot 6t^5}{t^{12}} = \frac{3 - 6\ln(2t^3)}{t^7}$$

of

$$h(t) = \frac{\ln(2t^3)}{t^6} = \frac{\ln(2) + \ln(t^3)}{t^6} = \frac{\ln(2) + 3\ln t}{t^6} \Rightarrow$$

$$h(t) = \frac{t^6 \cdot \left(0 + \frac{3}{t}\right) - \left(\ln(2) + 3\ln t\right) \cdot 6t^5}{t^{12}} = \frac{3t^5 - \left(\ln(2) + 3\ln t\right) \cdot 6t^5}{t^{12}}$$

$$= \frac{3 - \left(\ln(2) + 3\ln t\right) \cdot 6}{t^7} = \frac{3 - 6\ln(2) - 18\ln t}{t^7}$$

29.

a. $g(t) = \arccos(3t) - \arcsin(3t) \Rightarrow$

$$g'(t) = \frac{-1}{\sqrt{1-(3t)^2}} \cdot 3 - \frac{1}{\sqrt{1-(3t)^2}} \cdot 3 = \frac{-3}{\sqrt{1-9t^2}} - \frac{3}{\sqrt{1-9t^2}} = \frac{-6}{\sqrt{1-9t^2}}$$

b. $f(x) = \arccos\left(x^3\right) \Rightarrow f'(x) = \dfrac{-1}{\sqrt{1-\left(x^3\right)^2}} \cdot 3x^2 = \dfrac{-3x^2}{\sqrt{1-x^6}}$

c. $k(t) = t\arccos\left(\sqrt{1-t^2}\right) \Rightarrow$

$$k'(t) = 1\cdot\arccos\left(\sqrt{1-t^2}\right) + t\cdot\frac{1}{\sqrt{1-\left(\sqrt{1-t^2}\right)^2}}\cdot\frac{1}{2\sqrt{1-t^2}}\cdot(0-2t)$$

$$= \arccos\left(\sqrt{1-t^2}\right) - \frac{t^2}{\sqrt{1-\left(1-t^2\right)}\sqrt{1-t^2}}$$

$$= \arccos\left(\sqrt{1-t^2}\right) - \frac{t^2}{\sqrt{t^2}\sqrt{1-t^2}}$$

$$= \arccos\left(\sqrt{1-t^2}\right) - \frac{t^2}{|t|\sqrt{1-t^2}} = \arccos\left(\sqrt{1-t^2}\right) - \frac{|t|}{\sqrt{1-t^2}}$$

30.

a.
$$g(u) = 2\arctan\left(\tfrac{5}{u}\right) + 5\ln\left(u^2+25\right) \Rightarrow$$

$$g(u) = 2\cdot\frac{1}{1+\left(\frac{5}{u}\right)^2}\cdot\left(-\frac{5}{u^2}\right) + 5\cdot\frac{1}{u^2+25}\cdot 2u$$

$$= \frac{-10}{u^2+25} + \frac{10u}{u^2+25} = \frac{10u-10}{u^2+25}$$

b.
$$f(t) = \left(t+\arctan(3t)\right)^2 \Rightarrow$$

$$f'(t) = 2\cdot\left(t+\arctan(3t)\right)\cdot\left(1+\frac{1}{1+(3t)^2}\cdot 3\right)$$

$$= 2\cdot\left(t+\arctan(3t)\right)\cdot\left(1+\frac{3}{1+9t^2}\right)$$

$$= 2\cdot\left(t+\arctan(3t)\right)\cdot\frac{9t^2+4}{1+9t^2} = \frac{18t^2+8}{1+9t^2}\cdot\left(t+\arctan(3t)\right)$$

c.
$$g(x) = \frac{x^2+1}{\arctan x} \Rightarrow$$

$$g'(x) = \frac{\arctan x\cdot(2x+0) - \left(x^2+1\right)\cdot\frac{1}{x^2+1}}{(\arctan x)^2} = \frac{2x\arctan x - 1}{(\arctan x)^2}$$

31.

a.
$$y = \tfrac{1}{4}x^8 - 5x^3 + 2x \Rightarrow$$

$$\mathrm{d}y = \mathrm{d}\left(\tfrac{1}{4}x^8 - 5x^3 + 2x\right) = \mathrm{d}\left(\tfrac{1}{4}x^8\right) - \mathrm{d}\left(5x^3\right) + \mathrm{d}(2x)$$

$$= 2x^7\mathrm{d}x - 15x^2\mathrm{d}x + 2\mathrm{d}x = \left(2x^7 - 15x^2 + 2\right)\mathrm{d}x \Rightarrow$$

$$\mathrm{d}x = \frac{1}{2x^7 - 15x^2 + 2}\mathrm{d}y$$

b.
$$x^4y^2 - x^2y^4 = 4x^3y^3 \Rightarrow$$

$$\mathrm{d}\left(x^4y^2 - x^2y^4\right) = \mathrm{d}\left(4x^3y^3\right) \Rightarrow$$

$$\mathrm{d}\left(x^4y^2\right) - \mathrm{d}\left(x^2y^4\right) = \mathrm{d}\left(4x^3y^3\right) \Rightarrow$$

$$\left(\mathrm{d}\left(x^4\right)y^2 + x^4\mathrm{d}\left(y^2\right)\right) - \left(\mathrm{d}\left(x^2\right)y^4 + x^2\mathrm{d}\left(y^4\right)\right) = \mathrm{d}\left(4x^3\right)y^3 + 4x^3\mathrm{d}\left(y^3\right) \Rightarrow$$

$$\left(4x^3y^2\mathrm{d}x + 2x^4y\mathrm{d}y\right) - \left(2xy^4\mathrm{d}x + 4x^2y^3\mathrm{d}y\right) = 12x^2y^3\mathrm{d}x + 12x^3y^2\mathrm{d}y \Rightarrow$$

$$4x^3y^2\mathrm{d}x - 2xy^4\mathrm{d}x - 12x^2y^3\mathrm{d}x = 12x^3y^2\mathrm{d}y - 2x^4y\mathrm{d}y + 4x^2y^3\mathrm{d}y \Rightarrow$$

$$2xy^2\left(2x^2 - y^2 - 6xy\right)\mathrm{d}x = 2x^2y\left(6xy - x^2 + 2y^2\right)\mathrm{d}y \Rightarrow$$

$$\mathrm{d}x = \frac{2x^2y\left(6xy - x^2 + 2y^2\right)}{2xy^2\left(2x^2 - y^2 - 6xy\right)}\mathrm{d}y \Rightarrow \mathrm{d}x = -\frac{x\left(x^2 - 6xy - 2y^2\right)}{y\left(2x^2 - 6xy - y^2\right)}\mathrm{d}y$$

c. $z^2 \cdot t = \sin\left(z^2 + t\right) \Rightarrow$

$$\mathrm{d}\left(z^2 \cdot t\right) = \mathrm{d}\left(\sin\left(z^2 + t\right)\right) \Rightarrow$$

$$\mathrm{d}\left(z^2\right) \cdot t + z^2 \cdot \mathrm{d}(t) = \cos\left(z^2 + t\right) \cdot \mathrm{d}\left(z^2 + t\right) \Rightarrow$$

$$2z\,\mathrm{d}z \cdot t + z^2 \cdot \mathrm{d}t = \cos\left(z^2 + t\right) \cdot \left(2z\mathrm{d}z + \mathrm{d}t\right) \Rightarrow$$

$$2zt\,\mathrm{d}z - 2z\cos\left(z^2 + t\right)\mathrm{d}z = \cos\left(z^2 + t\right)\mathrm{d}t - z^2\mathrm{d}t \Rightarrow$$

$$\left(2zt - 2z\cos\left(z^2 + t\right)\right)\mathrm{d}z = \left(\cos\left(z^2 + t\right) - z^2\right)\mathrm{d}t \Rightarrow$$

$$\mathrm{d}z = \frac{\cos\left(z^2 + t\right) - z^2}{2zt - 2z\cos\left(z^2 + t\right)}\mathrm{d}t$$

d. $5x^3 \cdot \log y + 3y^5 \cdot \log x = x^2\sqrt{x} \Rightarrow$

$$\mathrm{d}\left(5x^3\right) \cdot \log y + 5x^3 \cdot \mathrm{d}(\log y) + \mathrm{d}\left(3y^5\right) \cdot \log x + 3y^5 \cdot \mathrm{d}(\log x) = \mathrm{d}\left(x^{\frac{5}{2}}\right) \Rightarrow$$

$$15x^2\mathrm{d}x \cdot \log y + 5x^3 \cdot \frac{1}{y\ln 10}\mathrm{d}y + 15y^2\mathrm{d}y \cdot \log x + 3y^5\frac{1}{x\ln 10}\mathrm{d}x = \frac{5}{2}x^{\frac{3}{2}}\mathrm{d}x \Rightarrow$$

$$\left(\frac{5x^3}{y\ln 10} + 15y^2\log x\right)\mathrm{d}y = \left(\frac{5}{2}x^{\frac{3}{2}} - 15x^2\log y - \frac{3y^5}{x\ln 10}\right)\mathrm{d}x \Rightarrow$$

[linker- en rechterlid met $2xy\ln 10$ vermeningvuldigen]

$$\left(10x^4 + 30\ln 10 \cdot xy^3\log x\right)\mathrm{d}y = \left(5\ln 10 \cdot xyx\sqrt{x} - 30\ln 10 \cdot x^3y\log y - 6y^6\right)\mathrm{d}x \Rightarrow$$

$$\mathrm{d}y = \frac{10x^4 + 30 \cdot \ln 10 \cdot xy^3\log x}{5y \cdot \ln 10 \cdot x\sqrt{x} - 30 \cdot \ln 10 \cdot x^3y \cdot \log y - 6y^6}\mathrm{d}x$$

32.

a. $x^3 + 3x^2y + y^3 = 5 \Rightarrow$

$$\mathrm{d}\left(x^3 + 3x^2y + y^3\right) = \mathrm{d}(5) \Rightarrow$$

$$3x^2\mathrm{d}x + 6x\mathrm{d}x \cdot y + 3x^2\mathrm{d}y + 3y^2\mathrm{d}y = 0 \Rightarrow$$

$$3\left(x^2 + y^2\right)\mathrm{d}y = -3x(x + 2y)\mathrm{d}x \Rightarrow$$

$$\frac{dy}{dx}=\frac{-3x(x+2y)}{3(x^2+y^2)}=-\frac{x(x+2y)}{x^2+y^2}$$

b. $\frac{x^2}{y}-\frac{y}{x^2}=1 \Rightarrow$

$$d\left(\frac{x^2}{y}\right)-d\left(\frac{y}{x^2}\right)=d(1)\Rightarrow$$

$$\frac{y\cdot d(x^2)-x^2\cdot d(y)}{y^2}-\frac{x^2\cdot d(y)-y\cdot d(x^2)}{x^4}=0\Rightarrow$$

$$\frac{2xydx-x^2dy}{y^2}-\frac{x^2dy-2xydx}{x^4}=0\Rightarrow$$

$$\frac{x^4(2xydx-x^2dy)-y^2(x^2dy-2xydx)}{x^4y^2}=0\Rightarrow$$

$$2x^5ydx-x^6dy-x^2y^2dy+2xy^3dx=0\Rightarrow$$

$$-x^6dy-x^2y^2dy=-2x^5ydx-2xy^3dx\Rightarrow$$

$$x^2(x^4+y^2)dy=2xy(x^4+y^2)dx\Rightarrow$$

$$\frac{dy}{dx}=\frac{2xy(x^4+y^2)}{x^2(x^4+y^2)}=2\frac{y}{x}$$

33.

a. $x^2-y^2=6xy-10\Rightarrow$

$$d(x^2-y^2)=d(6xy-10)\Rightarrow$$

$$2xdx-2ydy=6dx\cdot y+6xdy-0\Rightarrow$$

$$-2ydy-6xdy=-2xdx+6ydx\Rightarrow$$

$$ydy+3xdy=xdx-3ydx\Rightarrow$$

$$(y+3x)dy=(x-3y)dx\Rightarrow$$

$$\frac{dy}{dx}=\frac{x-3y}{3x+y}$$

b. We controleren of $P(-3,19)$ op de kromme K ligt.

We vullen $x=-3$ en $y=19$ in de kromme $K: x^2-y^2=6xy-10$ in:

Geldt $(-3)^2-19^2=6\cdot(-3)\cdot 19-10$?

Oftewel geldt $9-361=-342-10$?

Ja, dit klopt, want $-352=-352$.

De richtingscoëfficiënt *a* van de raaklijn m in het punt $P(-3,19)$ aan de kromme K

is: $a=\left[\frac{dy}{dx}\right]_{(x,y)=(-3,19)}=\left[\frac{x-3y}{3x+y}\right]_{(x,y)=(-3,19)}=\frac{-3-3\cdot 19}{3\cdot(-3)+19}=\frac{-60}{10}=-6$.

De vergelijking van de raaklijn m is dus: $y=-6x+b$.

Deze lijn gaat door $P(-3,19)$.

Dus geldt: $19 = -6 \cdot (-3) + b \Rightarrow 19 = 18 + b \Rightarrow b = 1$.

De vergelijking van de gevraagde raaklijn *m* is dus: $y = -6x + 1$

c. Een raaklijn loopt horizontaal als de richtingscoëfficiënt gelijk is aan 0.

Dus moet gelden: $\frac{dy}{dx} = 0 \Rightarrow \frac{x - 3y}{3x + y} = 0 \Rightarrow x - 3y = 0 \Rightarrow x = 3y$

Het punt moet op de kromme K liggen; we vullen $x = 3y$ in de vergelijking van K in:

$(3y)^2 - y^2 = 6(3y)y - 10 \Rightarrow 9y^2 - y^2 = 18y^2 - 10 \Rightarrow$

$10y^2 = 10 \Rightarrow y^2 = 1 \Rightarrow y = 1 \vee y = -1$

Samen met $x = 3y$ geeft dit de punten: $(3,1)$ en $(-3,-1)$

d. Een verticale lijn loop 'oneindig' steil, de richtingscoëfficiënt is dus 'oneindig' (of '-oneindig').

Dan moet gelden: $\frac{dy}{dx} = \pm\infty$, oftewel $\frac{dx}{dy} = 0$. Dit geeft:

$$\frac{dx}{dy} = 0 \Rightarrow \frac{3x + y}{x - 3y} = 0 \Rightarrow 3x + y = 0 \Rightarrow y = -3x$$

Het punt moet op de K liggen; we vullen $y = -3x$ in de vergelijking van K in:

$x^2 - y^2 = 6xy - 10 \Rightarrow x^2 - (-3x)^2 = 6x(-3x) - 10 \Rightarrow$

$x^2 - 9x^2 = -18x^2 - 10 \Rightarrow 10x^2 = -10 \Rightarrow x^2 = -1$

Deze vergelijking heeft dus geen oplossingen. Er zijn dusgeen punten waarin de raaklijn verticaal loopt.

34.

a.

$$\lim_{u\to 0} \frac{2^{3u} - 2^{-u}}{\sin(5u)} = \left(\frac{1-1}{0}\right) \underset{\langle\text{l' Hôpital}\rangle}{=} \lim_{u\to 0} \frac{\frac{d}{du}\left(2^{3u} - 2^{-u}\right)}{\frac{d}{du}\left(\sin(5u)\right)}$$

$$= \lim_{u\to 0} \frac{2^{3u} \cdot \ln 2 \cdot 3 - 2^{-u} \cdot \ln 2 \cdot (-1)}{\cos(5u) \cdot 5}$$

$$= \lim_{u\to 0} \frac{3\ln 2 \cdot 2^{3u} + \ln 2 \cdot 2^{-u}}{5\cos(5u)} = \frac{3\ln 2 + \ln 2}{5} = \frac{4\ln 2}{5} = \tfrac{4}{5}\ln 2$$

b.

$$\lim_{x\to 0} \frac{\cos(2x) - 1}{x^2} = \left(\frac{1-1}{0} = \frac{0}{0}\right) \underset{\langle\text{l'Hôpital}\rangle}{=} \lim_{x\to 0} \frac{\frac{d}{dx}\left(\cos(2x) - 1\right)}{\frac{d}{dx}\left(x^2\right)} = \lim_{x\to 0} \frac{-2\sin(2x)}{2x}$$

$$= \lim_{x\to 0} \frac{-\sin(2x)}{x} = \left(\frac{0}{0}\right) \underset{\langle\text{l'Hôpital}\rangle}{=} \lim_{x\to 0} \frac{\frac{d}{dx}\left(-\sin(2x)\right)}{\frac{d}{dx}(x)}$$

$$= \lim_{x\to 0} \frac{-2\cos(2x)}{1} = -2$$

c.

$$\lim_{t\to-\infty}\left(t^2\cdot e^{3t}\right)=\left(-\infty\cdot 0\right)=\lim_{t\to-\infty}\frac{t^2}{e^{-3t}}=\left(\frac{\infty}{\infty}\right)\underset{\langle\text{l'Hôpital}\rangle}{=}\lim_{t\to-\infty}\frac{\frac{d}{dt}\left(t^2\right)}{\frac{d}{dt}\left(e^{-3t}\right)}$$

$$=\lim_{t\to-\infty}\frac{2t}{-3e^{-3t}}=\left(\frac{\infty}{-\infty}\right)\underset{\langle\text{l'Hôpital}\rangle}{=}\lim_{t\to-\infty}\frac{\frac{d}{dt}\left(2t\right)}{\frac{d}{dt}\left(-3e^{-3t}\right)}$$

$$=\lim_{t\to-\infty}\frac{2}{9e^{-3t}}=\left(\frac{2}{\infty}\right)=0$$

35.

a. $$\lim_{x\to2}\frac{x^6-64}{x-2}=\left(\frac{64-64}{2-2}=\frac{0}{0}\right)\underset{\langle\text{l'Hôpital}\rangle}{=}\lim_{x\to2}\frac{\frac{d}{dx}\left(x^6-64\right)}{\frac{d}{dx}\left(x-2\right)}=\lim_{x\to2}\frac{6x^5}{1}=192$$

b.

$$\lim_{v\to-\infty}\frac{3v^4-4v+5}{3v^3-2v^5}=\left(\frac{\pm\infty}{\pm\infty}\right)=\lim_{v\to-\infty}\frac{3\frac{v^4}{v^5}-4\frac{v}{v^5}+\frac{5}{v^5}}{3\frac{v^3}{v^5}-2\frac{v^5}{v^5}}$$

$$=\lim_{v\to-\infty}\frac{3\frac{1}{v}-4\frac{1}{v^4}+\frac{5}{v^5}}{3\frac{1}{v^2}-2}=\frac{0-0+0}{0-2}=0$$

Berekening kan ook met het meerder keren toepassen van de regel van l´Hôpital.

c. $$\lim_{t\to\infty}\frac{2+t^2\sqrt{t}}{1+3t^2}=\left(\frac{\pm\infty}{\pm\infty}\right)=\lim_{t\to\infty}\frac{\frac{2}{t^2}+\frac{t^2\sqrt{t}}{t^2}}{\frac{1}{t^2}+\frac{3t^2}{t^2}}=\lim_{t\to\infty}\frac{\frac{2}{t^2}+\sqrt{t}}{\frac{1}{t^2}+3}=\left(\frac{0+\infty}{0+3}\right)=\infty$$

Berekening kan ook met het meerder keren toepassen van de regel van l´Hôpital.

36.

a. $$\lim_{x\to2}\frac{x^2\sqrt{x+7}-ax+3}{x-2}=\left(\frac{4\sqrt{9}-2a+3}{2-2}=\frac{15-2a}{0}\right)$$

Deze limiet kan alleen als reëel getal bestaan als naast de noemer ook de teller gelijk is aan 0.

Dat betekent dat $15-2a=0$, dus: $a=\frac{15}{2}=7\frac{1}{2}$

b.

$$\lim_{x\to 2}\frac{x^2\sqrt{x+7}-ax+3}{x-2} = \left\langle \text{vul in: } a=\tfrac{15}{2}\right\rangle = \lim_{x\to 2}\frac{x^2\sqrt{x+7}-\tfrac{15}{2}x+3}{x-2}$$

$$= \left(\frac{0}{0}\right) \underset{\langle \text{l'Hôpital}\rangle}{=} \lim_{x\to 2}\frac{\frac{d}{dx}\left(x^2\sqrt{x+7}-\tfrac{15}{2}x+3\right)}{\frac{d}{dx}\left(x-2\right)}$$

$$= \lim_{x\to 2}\frac{2x\sqrt{x+7}+x^2\dfrac{1}{2\sqrt{x+7}}-\dfrac{15}{2}}{1}$$

$$= \lim_{x\to 2} 2x\sqrt{x+7}+x^2\frac{1}{2\sqrt{x+7}}-\frac{15}{2}$$

$$= 4\sqrt{9}+4\cdot\frac{1}{2\sqrt{9}}-\frac{15}{2} = 12+\frac{2}{3}-\frac{15}{2} = \frac{31}{6} = 5\tfrac{1}{6}$$