



Uitwerkingen extra opgaven hoofdstuk 1 Basisvaardigheden en basistechnieken

1.

a.

$$\begin{aligned}3\frac{1}{3} - 2\frac{1}{6} + \left(\frac{1}{2}\right)^3 + \left(2\frac{1}{3}\right)^2 &= 3\frac{1}{3} - 2\frac{1}{6} + \frac{1}{8} + \left(\frac{7}{3}\right)^2 \\&= 3\frac{1}{3} - 2\frac{1}{6} + \frac{1}{8} + \frac{49}{9} \\&= 3\frac{24}{72} - 2\frac{12}{72} + \frac{9}{72} + \frac{392}{72} \\&= 1\frac{12}{72} + \frac{401}{72} \\&= \frac{84}{72} + \frac{401}{72} \\&= \frac{485}{72} \\&= 6\frac{53}{72}\end{aligned}$$

b.

$$\begin{aligned}3\sqrt{2} - \sqrt{8} + \sqrt{\frac{1}{2}} + \frac{3}{\sqrt{2}} &= 3\sqrt{2} - \sqrt{2 \cdot 4} + \sqrt{\frac{1}{2}} + \frac{3 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} \\&= 3\sqrt{2} - 2\sqrt{2} + \frac{\sqrt{1} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} + \frac{3\sqrt{2}}{2} \\&= \sqrt{2} + \frac{\sqrt{2}}{2} + \frac{3\sqrt{2}}{2} \\&= \sqrt{2} + \frac{4\sqrt{2}}{2} \\&= \sqrt{2} + 2\sqrt{2} \\&= 3\sqrt{2}\end{aligned}$$

c.

$$\begin{aligned}\frac{\frac{1}{4} + \frac{1}{3}}{\frac{7}{12} - \frac{1}{6}} &= \frac{\frac{3}{12} + \frac{4}{12}}{\frac{7}{12} - \frac{2}{12}} \\&= \frac{\frac{7}{12}}{\frac{5}{12}} \\&= \frac{7}{12} \cdot \frac{12}{5} \\&= \frac{7}{5} \\&= 1\frac{2}{5}\end{aligned}$$

d.

$$\begin{aligned}\left(2\frac{3}{4} - 1\frac{2}{3}\right)\left(3\frac{1}{2} - 3\frac{1}{3}\right) &= \left(2\frac{9}{12} - 1\frac{8}{12}\right)\left(3\frac{3}{6} - 3\frac{2}{6}\right) \\&= 1\frac{1}{12} \cdot \frac{1}{6} \\&= \frac{13}{12} \cdot \frac{1}{6} \\&= \frac{13}{72}\end{aligned}$$



2.

a.

$$(a-b)^2 - (a-b)(a+b) = a^2 - 2ab + b^2 - (a^2 - b^2) \\ = 2b^2 - 2ab$$

b.

$$x(x-3) + (x-1)(x+2) - 2(x-3)(x+2) = x^2 - 3x + x^2 + x - 2 - 2(x^2 - x - 6) \\ = 2x^2 - 2x - 2 - 2x^2 + 2x + 12 \\ = 10$$

c.

$$(a+b)^3 - (a-b)^3 = (a+b)(a^2 + 2ab + b^2) - (a-b)(a^2 - 2ab + b^2) \\ = a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3 - (a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3) \\ = a^3 + 3a^2b + 3ab^2 + b^3 - (a^3 - 3a^2b + 3ab^2 - b^3) \\ = 6a^2b + 2b^3$$

d.

$$x(x^2 - 3x)(x-2) = (x^3 - 3x^2)(x-2) \\ = x^4 - 2x^3 - 3x^3 + 6x^2 \\ = x^4 - 5x^3 + 6x^2$$

3.

a.

$$(x^2 - 2x - 3)(x^2 + 2x + 1) = (x-3)(x+1)(x+1)^2 \\ = (x-3)(x+1)^3$$

b.

$$4x^3 - 8x^2 - 32x = 4x(x^2 - 2x - 8) \\ = 4x(x-4)(x+2)$$

c.

$$p^2 - 2pq + q^2 = (p-q)^2$$

d.

$$4x^4 + 8x^2 - 12 = 4(x^4 + 2x^2 - 3) \\ = 4(x^2 + 3)(x^2 - 1) \\ = 4(x^2 + 3)(x-1)(x+1)$$

4.

a.

$$\frac{x+a}{b} = \frac{x-a}{c} \Rightarrow c(x+a) = b(x-a) \\ \Rightarrow cx + ac = bx - ab \\ \Rightarrow (c-b)x = -ac - ab \\ \Rightarrow x = \frac{-ac - ab}{c-b} = \frac{ac + ab}{b-c} = \frac{a(b+c)}{b-c}$$



b.

$$\begin{aligned}\frac{a}{x} &= \frac{b+c}{a} - \frac{1}{a^2} \Rightarrow \frac{a}{x} = \frac{a(b+c)}{a^2} - \frac{1}{a^2} = \frac{ab+ac-1}{a^2} \\ &\Rightarrow \frac{x}{a} = \frac{a^2}{ab+ac-1} \\ &\Rightarrow x = \frac{a^3}{ab+ac-1}\end{aligned}$$

c.

$$\begin{aligned}ax - bc &= bx + bc \Rightarrow ax - bx = 2bc \\ &\Rightarrow (a-b)x = 2bc \\ &\Rightarrow x = \frac{2bc}{a-b}\end{aligned}$$

d.

$$\begin{aligned}\frac{x}{x+a} &= b \Rightarrow x = bx + ab \\ &\Rightarrow x - bx = ab \\ &\Rightarrow x(1-b) = ab \\ &\Rightarrow x = \frac{ab}{1-b}\end{aligned}$$

5.

a.

$$3x - 8 = 5x - 3 \Rightarrow -2x = 5 \Rightarrow x = -\frac{5}{2}$$

b.

$$x^2 = 6x + 7 \Rightarrow x^2 - 6x - 7 = 0 \Rightarrow (x-7)(x+1) = 0 \Rightarrow x = 7 \vee x = -1$$

c.

$$\begin{aligned}\sqrt{x} &= x - 2 \Rightarrow x = (x-2)^2 \\ &\Rightarrow x = x^2 - 4x + 4 \\ &\Rightarrow x^2 - 5x + 4 = 0 \\ &\Rightarrow (x-1)(x-4) = 0 \\ &\Rightarrow x = 1 \vee x = 4\end{aligned}$$

controle: alleen $x = 4$ voldoet

d.

$$\begin{aligned}\sqrt{x-1} + x &= 7 \Rightarrow \sqrt{x-1} = 7-x \\ &\Rightarrow x-1 = (7-x)^2 \\ &\Rightarrow x-1 = 49 - 14x + x^2 \\ &\Rightarrow x^2 - 15x + 50 = 0 \\ &\Rightarrow (x-5)(x-10) = 0 \\ &\Rightarrow x = 5 \vee x = 10\end{aligned}$$

controle: alleen $x = 5$ voldoet



6.

a.

$$\begin{cases} 3x - 2y = 10 \\ x + y = 5 \end{cases} \Rightarrow \begin{cases} 3x - 2y = 10 \\ 3x + 3y = 15 \end{cases} \Rightarrow -5y = -5 \Rightarrow y = 1 \Rightarrow x = 4$$

b.

$$\begin{cases} 3x - ay = b \\ x + cy = d \end{cases} \Rightarrow \begin{cases} 3x - ay = b \\ 3x + 3cy = 3d \end{cases} \Rightarrow y(-a - 3c) = b - 3d \Rightarrow y = \frac{b - 3d}{-a - 3c} = \frac{3d - b}{a + 3c}$$

$$\Rightarrow x = d - c \frac{3d - b}{a + 3c} = \frac{da + 3cd - 3cd + cb}{a + 3c} = \frac{da + cb}{a + 3c} \text{ (op voorwaarde dat } a + 3c \neq 0)$$

7.

a.

$$\frac{(a^2 b^3 c^4)^2}{abc} = \frac{a^4 b^6 c^8}{abc} = a^3 b^5 c^7$$

b.

$$\frac{\sqrt[3]{a} \sqrt{b} \sqrt[4]{c^7}}{\sqrt[3]{a^2} \sqrt[4]{b^3} \sqrt[5]{c^5}} = \frac{a^{\frac{1}{3}} b^{\frac{1}{2}} c^{\frac{7}{4}}}{a^{\frac{2}{3}} b^{\frac{3}{4}} c^{\frac{5}{4}}} = a^{\frac{1}{3} - \frac{2}{3}} b^{\frac{1}{2} - \frac{3}{4}} c^{\frac{7}{4} - \frac{5}{4}} = a^{-\frac{1}{3}} b^{-\frac{1}{4}} c^{\frac{2}{4}} = a^{-\frac{1}{3}} b^{-\frac{1}{4}} c^{\frac{1}{2}}$$

c.

$$\frac{(x+1)^3 (x-1)^2}{x^2 - 1} = \frac{(x+1)^3 (x-1)^2}{(x+1)(x-1)} = (x+1)^2 (x-1)$$

d.

$$\frac{x(x+1) - 2(x+3)}{(x-3)^2} = \frac{x^2 + x - 2x - 6}{(x-3)^2} = \frac{x^2 - x - 6}{(x-3)^2} = \frac{(x-3)(x+2)}{(x-3)^2} = \frac{x+2}{x-3}$$

8.

a.

$$x^3 = 2 \Rightarrow x = \sqrt[3]{2} = 2^{\frac{1}{3}} \approx 1,26$$

b.

$$x\sqrt{x} = 8 \Rightarrow x^{\frac{3}{2}} = 8 \Rightarrow x = 8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

c.

$$x^{\frac{1}{5}} = 2 \Rightarrow x = 2^5 = 32$$

d.

$$x^{1,5} = 3 \Rightarrow x^{\frac{3}{2}} = 3 \Rightarrow x = 3^{\frac{2}{3}} \approx 2,08$$

9.

a.

$${}^4 \log x^2 = 2 \Rightarrow x^2 = 4^2 = 16 \Rightarrow x = 4 \vee x = -4$$

$$\text{N.B. Niet } {}^4 \log x^2 = 2 \Rightarrow 2 \cdot {}^4 \log x = 2 \Rightarrow {}^4 \log x = 1 \Rightarrow x = 4$$

want hier kan x ook negatief zijn

b.

$$({}^4 \log x)^2 = 2 \Rightarrow {}^4 \log x = \pm \sqrt{2} \Rightarrow x = 4^{\sqrt{2}} \approx 7,103 \vee x = 4^{-\sqrt{2}} \approx 0,141$$



c.

$$\log x + \log(2x) = 2 \Rightarrow \log(2x^2) = 2 \Rightarrow 2x^2 = 10^2 = 100 \Rightarrow x^2 = 50 \Rightarrow x = \pm\sqrt{50}$$

Hier vervalt de oplossing $x = -\sqrt{50}$ want x moet positief zijn, dus $x = \sqrt{50} \approx 7,07$

d.

$$\frac{1}{4} \log x = -0,5 \Rightarrow x = \left(\frac{1}{4}\right)^{-0,5} = \frac{1}{\left(\frac{1}{4}\right)^{0,5}} = \frac{1}{\sqrt{\frac{1}{4}}} = \frac{1}{\frac{1}{2}} = 2$$

10.

a.

$${}^2 \log y = x \Rightarrow y = 2^x$$

b.

$$\log(2y) = 2 \log x \Rightarrow \log(2y) = \log(x^2) \Rightarrow 2y = x^2 \Rightarrow y = \frac{1}{2} x^2$$

c.

$${}^3 \log y = {}^2 \log x \Rightarrow \frac{\log y}{\log 3} = \frac{\log x}{\log 2} \Rightarrow \log y = \frac{\log 3 \cdot \log x}{\log 2} = {}^2 \log 3 \cdot \log x$$

$$\Rightarrow y = 10^{2 \log 3 \cdot \log x} = \left(10^{2 \log 3}\right)^{\log x}$$

$$\text{Eenvoudiger: } {}^3 \log y = {}^2 \log x \Rightarrow y = 3^{2 \log x}$$

d.

$$\log(y^3) = 3^2 \log x \Rightarrow 3 \log y = 3^2 \log x \Rightarrow \log y = {}^2 \log x \Rightarrow y = 10^{2 \log x}$$

11.

a.

$$|2x| = x + 1$$

$$x \geq 0: 2x = x + 1 \Rightarrow x = 1 \text{ (voldoet)}$$

$$x < 0: -2x = x + 1 \Rightarrow -3x = 1 \Rightarrow x = -\frac{1}{3} \text{ (voldoet)}$$

b.

$$|x| + |3x - 1| = 5$$

$$x \geq \frac{1}{3}: x + (3x - 1) = 5 \Rightarrow 4x = 6 \Rightarrow x = 1\frac{1}{2}$$

$$0 \leq x < \frac{1}{3}: x - (3x - 1) = 5 \Rightarrow -2x = 4 \Rightarrow x = -2 \text{ (vervalt dus)}$$

$$x < 0: -x - (3x - 1) = 5 \Rightarrow -4x = 4 \Rightarrow x = -1$$

$$\text{Dus er zijn twee oplossingen: } x = 1\frac{1}{2} \vee x = -1$$

c.

$$x^2 - 2|x| = 3$$

$$x \geq 0: x^2 - 2x = 3 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x - 3)(x + 1) = 0 \Rightarrow x = 3 \vee x = -1 \text{ (vervalt)}$$

$$x < 0: x^2 + 2x = 3 \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow (x + 3)(x - 1) = 0 \Rightarrow x = -3 \vee x = 1 \text{ (vervalt)}$$

$$\text{Dus: } x = 3 \vee x = -3$$



d.

$$|x^2 - 2x - 8| = 7 \Rightarrow |(x-4)(x+2)| = 7$$

$$x \geq 4: (x-4)(x+2) = 7 \Rightarrow x^2 - 2x - 8 = 7 \Rightarrow x^2 - 2x - 15 = 0$$

$$\Rightarrow (x-5)(x+3) = 0 \Rightarrow x = 5 \text{ (voldoet)} \vee x = -3 \text{ (vervalt)}$$

$$-2 \leq x < 4: -(x-4)(x+2) = 7 \Rightarrow x^2 - 2x - 8 = -7 \Rightarrow x^2 - 2x - 1 = 0$$

$$\Rightarrow x = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2} \text{ (voldoen beide)}$$

$$x < -2: (x-4)(x+2) = 7 \Rightarrow x = 5 \text{ (vervalt)} \vee x = -3 \text{ (voldoet)}$$

$$\text{Dus: } x = -3 \vee x = 1 - \sqrt{2} \approx -0,41 \vee x = 1 + \sqrt{2} \approx 2,41 \vee x = 5$$

12.

$$T_F = \frac{9}{5}T_C + 32 \Rightarrow \frac{9}{5}T_C = T_F - 32 \Rightarrow T_C = \frac{5}{9}(T_F - 32)$$

13.

$$\frac{1}{a+b\sqrt{c}} = \frac{a-b\sqrt{c}}{(a+b\sqrt{c})(a-b\sqrt{c})} = \frac{a-b\sqrt{c}}{a^2 - (b\sqrt{c})^2} = \frac{a-b\sqrt{c}}{a^2 - b^2c}$$

$$\frac{1}{1+\sqrt{2}} = \frac{1-\sqrt{2}}{1-2} = -(1-\sqrt{2}) = \sqrt{2} - 1$$