Uitwerkingen extra opgaven hoofdstuk 1 Basisvaardigheden, basistechnieken

1.

a.

$$3\tfrac{1}{3}-2\tfrac{1}{6}+\left(\tfrac{1}{2}\right)^3+\left(2\tfrac{1}{3}\right)^2 = 3\tfrac{1}{3}-2\tfrac{1}{6}+\tfrac{1}{8}+\left(\tfrac{7}{3}\right)^2$$
$$= 3\tfrac{1}{3}-2\tfrac{1}{6}+\tfrac{1}{8}+\tfrac{49}{9}$$
$$= 3\tfrac{24}{72}-2\tfrac{12}{72}+\tfrac{9}{72}+\tfrac{392}{72}$$
$$= 1\tfrac{12}{72}+\tfrac{401}{72}$$
$$= \tfrac{84}{72}+\tfrac{401}{72}$$
$$= \tfrac{485}{72}$$
$$= 6\tfrac{53}{72}$$

b.

$$3\sqrt{2}-\sqrt{8}+\sqrt{\tfrac{1}{2}}+\frac{3}{\sqrt{2}} = 3\sqrt{2}-\sqrt{2\cdot 4}+\sqrt{\frac{1}{2}}+\frac{3\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$$
$$= 3\sqrt{2}-2\sqrt{2}+\frac{\sqrt{1}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}+\frac{3\sqrt{2}}{2}$$
$$= \sqrt{2}+\frac{\sqrt{2}}{2}+\frac{3\sqrt{2}}{2}$$
$$= \sqrt{2}+\frac{4\sqrt{2}}{2}$$
$$= \sqrt{2}+2\sqrt{2}$$
$$= 3\sqrt{2}$$

c.

$$\frac{\frac{1}{4}+\frac{1}{3}}{\frac{7}{12}-\frac{1}{6}} = \frac{\frac{3}{12}+\frac{4}{12}}{\frac{7}{12}-\frac{2}{12}}$$
$$= \frac{\frac{7}{12}}{\frac{5}{12}}$$
$$= \tfrac{7}{12}\cdot\tfrac{12}{5}$$
$$= \tfrac{7}{5}$$
$$= 1\tfrac{2}{5}$$

d.

$$\left(2\tfrac{3}{4}-1\tfrac{2}{3}\right)\left(3\tfrac{1}{2}-3\tfrac{1}{3}\right) = \left(2\tfrac{9}{12}-1\tfrac{8}{12}\right)\left(3\tfrac{3}{6}-3\tfrac{2}{6}\right)$$
$$= 1\tfrac{1}{12}\cdot\tfrac{1}{6}$$
$$= \tfrac{13}{12}\cdot\tfrac{1}{6}$$
$$= \tfrac{13}{72}$$

2.

a.

$$(a-b)^2-(a-b)(a+b)=a^2-2ab+b^2-(a^2-b^2)$$
$$=2b^2-2ab$$

b.

$$x(x-3)+(x-1)(x+2)-2(x-3)(x+2)=x^2-3x+x^2+x-2-2(x^2-x-6)$$
$$=2x^2-2x-2-2x^2+2x+12$$
$$=10$$

c.

$$(a+b)^3-(a-b)^3=(a+b)(a^2+2ab+b^2)-(a-b)(a^2-2ab+b^2)$$
$$=a^3+2a^2b+ab^2+a^2b+2ab^2+b^3-(a^3-2a^2b+ab^2-a^2b+2ab^2-b^3)$$
$$=a^3+3a^2b+3ab^2+b^3-(a^3-3a^2b+3ab^2-b^3)$$
$$=6a^2b+2b^3$$

d.

$$x\left(x^2-3x\right)(x-2)=(x^3-3x^2)(x-2)$$
$$=x^4-2x^3-3x^3+6x^2$$
$$=x^4-5x^3+6x^2$$

3.

a.

$$(x^2-2x-3)(x^2+2x+1)=(x-3)(x+1)(x+1)^2$$
$$=(x-3)(x+1)^3$$

b.

$$4x^3-8x^2-32x=4x(x^2-2x-8)$$
$$=4x(x-4)(x+2)$$

c.

$$p^2-2pq+q^2=(p-q)^2$$

d.

$$4x^4+8x^2-12=4(x^4+2x^2-3)$$
$$=4(x^2+3)(x^2-1)$$
$$=4(x^2+3)(x-1)(x+1)$$

4.

a.

$$\frac{x+a}{b}=\frac{x-a}{c}\Rightarrow c(x+a)=b(x-a)$$
$$\Rightarrow cx+ac=bx-ab$$
$$\Rightarrow (c-b)x=-ac-ab$$
$$\Rightarrow x=\frac{-ac-ab}{c-b}=\frac{ac+ab}{b-c}=\frac{a(b+c)}{b-c}$$

b.

$$\frac{a}{x}=\frac{b+c}{a}-\frac{1}{a^2}\Rightarrow\frac{a}{x}=\frac{a(b+c)}{a^2}-\frac{1}{a^2}=\frac{ab+ac-1}{a^2}$$

$$\Rightarrow\frac{x}{a}=\frac{a^2}{ab+ac-1}$$

$$\Rightarrow x=\frac{a^3}{ab+ac-1}$$

c.

$$ax-bc=bx+bc\Rightarrow ax-bx=2bc$$

$$\Rightarrow(a-b)x=2bc$$

$$\Rightarrow x=\frac{2bc}{a-b}$$

d.

$$\frac{x}{x+a}=b\Rightarrow x=bx+ab$$

$$\Rightarrow x-bx=ab$$

$$\Rightarrow x(1-b)=ab$$

$$\Rightarrow x=\frac{ab}{1-b}$$

5.

a.

$$3x-8=5x-3\Rightarrow -2x=5\Rightarrow x=-\tfrac{5}{2}$$

b.

$$x^2=6x+7\Rightarrow x^2-6x-7=0\Rightarrow(x-7)(x+1)=0\Rightarrow x=7\vee x=-1$$

c.

$$\sqrt{x}=x-2\Rightarrow x=(x-2)^2$$

$$\Rightarrow x=x^2-4x+4$$

$$\Rightarrow x^2-5x+4=0$$

$$\Rightarrow(x-1)(x-4)=0$$

$$\Rightarrow x=1\vee x=4$$

controle: alleen $x=4$ voldoet

d.

$$\sqrt{x-1}+x=7\Rightarrow\sqrt{x-1}=7-x$$

$$\Rightarrow x-1=(7-x)^2$$

$$\Rightarrow x-1=49-14x+x^2$$

$$\Rightarrow x^2-15x+50=0$$

$$\Rightarrow(x-5)(x-10)=0$$

$$\Rightarrow x=5\vee x=10$$

controle: alleen $x=5$ voldoet

6.

a.

$$\begin{cases} 3x-2y=10 \\ x+y=5 \end{cases} \Rightarrow \begin{cases} 3x-2y=10 \\ 3x+3y=15 \end{cases} \Rightarrow -5y=-5 \Rightarrow y=1 \Rightarrow x=4$$

b.

$$\begin{cases} 3x-ay=b \\ x+cy=d \end{cases} \Rightarrow \begin{cases} 3x-ay=b \\ 3x+3cy=3d \end{cases} \Rightarrow y(-a-3c) = \Rightarrow y=\frac{b-3d}{-a-3c}=\frac{3d-b}{a+3c}$$

$$\Rightarrow x=d-c\frac{3d-b}{a+3c}=\frac{da+3cd-3cd+cb}{a+3c}=\frac{da+cb}{a+3c} \text{ (op voorwaarde dat } a+3c\neq 0)$$

7.

a.

$$\frac{\left(a^2b^3c^4\right)^2}{abc}=\frac{a^4b^6c^8}{abc}=a^3b^5c^7$$

b.

$$\frac{\sqrt[3]{a}\sqrt{b}\sqrt[4]{c^7}}{\sqrt[3]{a^2}\sqrt[4]{b^3c^5}}=\frac{a^{\frac{1}{3}}b^{\frac{1}{2}}c^{\frac{7}{4}}}{a^{\frac{2}{3}}b^{\frac{3}{4}}c^{\frac{5}{4}}}=a^{\frac{1}{3}-\frac{2}{3}}b^{\frac{1}{2}-\frac{3}{4}}c^{\frac{7}{4}-\frac{5}{4}}=a^{-\frac{1}{3}}b^{-\frac{1}{4}}c^{\frac{2}{4}}=a^{-\frac{1}{3}}b^{-\frac{1}{4}}c^{\frac{1}{2}}$$

c.

$$\frac{(x+1)^3(x-1)^2}{x^2-1}=\frac{(x+1)^3(x-1)^2}{(x+1)(x-1)}=(x+1)^2(x-1)$$

d.

$$\frac{x(x+1)-2(x+3)}{(x-3)^2}=\frac{x^2+x-2x-6}{(x-3)^2}=\frac{x^2-x-6}{(x-3)^2}=\frac{(x-3)(x+2)}{(x-3)^2}=\frac{x+2}{x-3}$$

8.

a.

$$x^3=2 \Rightarrow x=\sqrt[3]{2}=2^{\frac{1}{3}}\approx 1,26$$

b.

$$x\sqrt{x}=8 \Rightarrow x^{\frac{3}{2}}=8 \Rightarrow x=8^{\frac{2}{3}}=\sqrt[3]{8^2}=\sqrt[3]{64}=4$$

c.

$$x^{\frac{1}{5}}=2 \Rightarrow x=2^5=32$$

d.

$$x^{1,5}=3 \Rightarrow x^{\frac{3}{2}}=3 \Rightarrow x=3^{\frac{2}{3}}\approx 2,08$$

9.

a.

$${}^4\log x^2=2 \Rightarrow x^2=4^2=16 \Rightarrow x=4 \vee x=-4$$

N.B. Niet ${}^4\log x^2=2 \Rightarrow 2\cdot {}^4\log x=2 \Rightarrow {}^4\log x=1 \Rightarrow x=4$

want hier kan x ook negatief zijn

b.

$$\left({}^4\log x\right)^2=2 \Rightarrow {}^4\log x=\pm\sqrt{2} \Rightarrow x=4^{\sqrt{2}}\approx 7,103 \vee x=4^{-\sqrt{2}}\approx 0,141$$

c.

$\log x + \log(2x) = 2 \Rightarrow \log(2x^2) = 2 \Rightarrow 2x^2 = 10^2 = 100 \Rightarrow x^2 = 50 \Rightarrow x = \pm\sqrt{50}$

Hier vervalt de oplossing $x = -\sqrt{50}$ want x moet positief zijn, dus $x = \sqrt{50} \approx 7,07$

d.

$^{\frac{1}{4}}\log x = -0,5 \Rightarrow x = \left(\frac{1}{4}\right)^{-0,5} = \frac{1}{\left(\frac{1}{4}\right)^{0,5}} = \frac{1}{\sqrt{\frac{1}{4}}} = \frac{1}{\frac{1}{2}} = 2$

10.

a.

$^{2}\log y = x \Rightarrow y = 2^x$

b.

$\log(2y) = 2\log x \Rightarrow \log(2y) = \log\left(x^2\right) \Rightarrow 2y = x^2 \Rightarrow y = \frac{1}{2}x^2$

c.

$^{3}\log y = {}^{2}\log x \Rightarrow \frac{\log y}{\log 3} = \frac{\log x}{\log 2} \Rightarrow \log y = \frac{\log 3 \cdot \log x}{\log 2} = {}^{2}\log 3 \cdot \log x$

$\Rightarrow y = 10^{{}^{2}\log 3 \cdot \log x} = \left(10^{{}^{2}\log 3}\right)^{\log x}$

Eenvoudiger: $^{3}\log y = {}^{2}\log x \Rightarrow y = 3^{{}^{2}\log x}$

d.

$\log(y^3) = 3\,{}^{2}\log x \Rightarrow 3\log y = 3\,{}^{2}\log x \Rightarrow \log y = {}^{2}\log x \Rightarrow y = 10^{{}^{2}\log x}$

11.

a.

$|2x| = x + 1$

$x \geq 0 : 2x = x + 1 \Rightarrow x = 1$ (voldoet)

$x < 0 : -2x = x + 1 \Rightarrow -3x = 1 \Rightarrow x = -\frac{1}{3}$ (voldoet)

b.

$|x| + |3x - 1| = 5$

$x \geq \frac{1}{3} : x + (3x - 1) = 5 \Rightarrow 4x = 6 \Rightarrow x = 1\frac{1}{2}$

$0 \leq x < \frac{1}{3} : x - (3x - 1) = 5 \Rightarrow -2x = 4 \Rightarrow x = -2$ (vervalt dus)

$x < 0 : -x - (3x - 1) = 5 \Rightarrow -4x = 4 \Rightarrow x = -1$

Dus er zijn twee oplossingen: $x = 1\frac{1}{2} \vee x = -1$

c.

$x^2 - 2|x| = 3$

$x \geq 0 : x^2 - 2x = 3 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x - 3)(x + 1) = 0 \Rightarrow x = 3 \vee x = -1$ (vervalt)

$x < 0 : x^2 + 2x = 3 \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow (x + 3)(x - 1) = 0 \Rightarrow x = -3 \vee x = 1$ (vervalt)

Dus: $x = 3 \vee x = -3$

d.

$|x^2-2x-8|=7 \Rightarrow |(x-4)(x+2)|=7$

$x \geq 4 : (x-4)(x+2)=7 \Rightarrow x^2-2x-8=7 \Rightarrow x^2-2x-15=0$

$\Rightarrow (x-5)(x+3)=0 \Rightarrow x=5 \text{ (voldoet)} \vee x=-3 \text{ (vervalt)}$

$-2 \leq x < 4 : -(x-4)(x+2)=7 \Rightarrow x^2-2x-8=-7 \Rightarrow x^2-2x-1=0$

$\Rightarrow x=\frac{2 \pm \sqrt{4+4}}{2}=1 \pm \sqrt{2} \text{ (voldoen beide)}$

$x < -2 : (x-4)(x+2)=7 \Rightarrow x=5 \text{ (vervalt)} \vee x=-3 \text{ (voldoet)}$

Dus: $x=-3 \vee x=1-\sqrt{2} \approx -0,41 \vee x=1+\sqrt{2} \approx 2,41 \vee x=5$

12.

$T_F=\frac{9}{5}T_C+32 \Rightarrow \frac{9}{5}T_C=T_F-32 \Rightarrow T_C=\frac{5}{9}(T_F-32)$

13.

$$\frac{1}{a+b\sqrt{c}}=\frac{a-b\sqrt{c}}{(a+b\sqrt{c})(a-b\sqrt{c})}=\frac{a-b\sqrt{c}}{a^2-(b\sqrt{c})^2}=\frac{a-b\sqrt{c}}{a^2-b^2c}$$

$$\frac{1}{1+\sqrt{2}}=\frac{1-\sqrt{2}}{1-2}=-(1-\sqrt{2})=\sqrt{2}-1$$