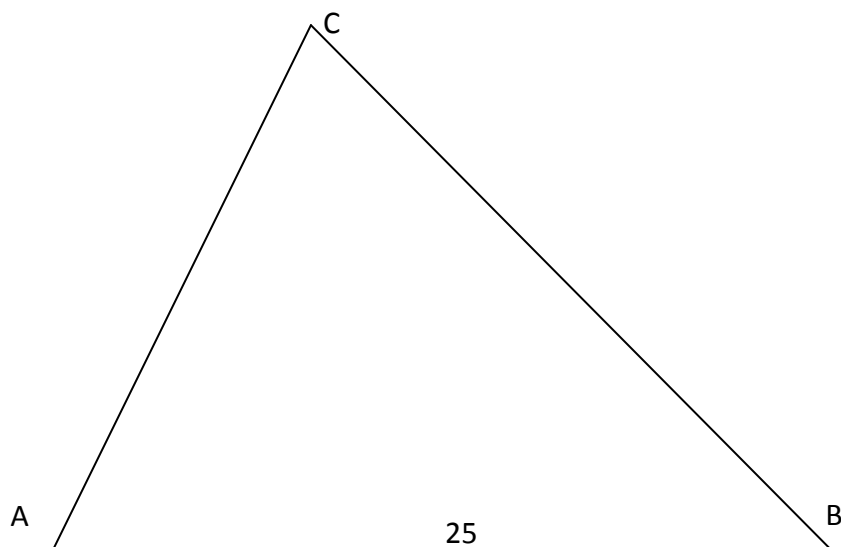




Uitwerkingen extra opgaven hoofdstuk 3 Goniometrie

1.



Noem $\angle A = \alpha$, $\angle B = \beta$ en $\angle C = \gamma$.

$$\alpha = 74^\circ, \beta = 46^\circ \Rightarrow \gamma = 180^\circ - 74^\circ - 46^\circ = 60^\circ .$$

Met de sinusregel berekenen we AC en BC:

$$\frac{\sin \alpha}{BC} = \frac{\sin \beta}{AC} = \frac{\sin \gamma}{AB} \Rightarrow \frac{\sin 74^\circ}{BC} = \frac{\sin 46^\circ}{AC} = \frac{\sin 60^\circ}{25} \approx 0,03464 .$$

$$\text{Hieruit volgt: } BC \approx \frac{\sin 74^\circ}{0,03464} \approx 27,75 \text{ en } AC \approx \frac{\sin 46^\circ}{0,03464} \approx 20,77$$

2.

Uit de gegevens volgt: $\angle BQP = 59^\circ + 33^\circ = 92^\circ$ en $\angle APQ = 70^\circ + 55^\circ = 125^\circ$.

We passen nu de sinusregel toe in driehoek BPQ om BQ te bepalen en we passen de sinusregel toe in driehoek APQ om AQ te bepalen:

$$\frac{\sin 55^\circ}{BQ} = \frac{\sin(\angle PBQ)}{PQ} = \frac{\sin(180^\circ - 55^\circ - 92^\circ)}{PQ} \Rightarrow BQ = PQ \cdot \frac{\sin 55^\circ}{\sin 33^\circ} \approx 400 \cdot \frac{0,81915}{0,54464} \approx 601,6$$

$$\frac{\sin 125^\circ}{AQ} = \frac{\sin(\angle PAQ)}{PQ} = \frac{\sin(180^\circ - 125^\circ - 33^\circ)}{PQ} \Rightarrow AQ = PQ \cdot \frac{\sin 125^\circ}{\sin 22^\circ} \approx 400 \cdot \frac{0,81915}{0,37461} \approx 874,7$$

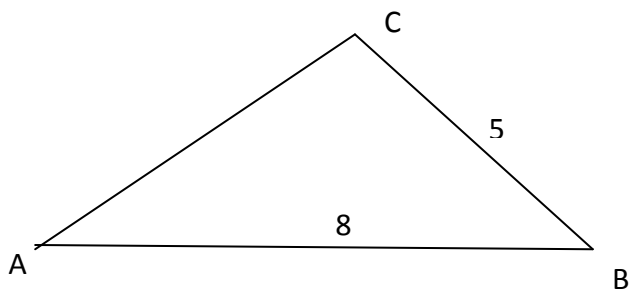
Met de cosinusregel bepalen we nu AB:

$$AB^2 = AQ^2 + BQ^2 - 2AQ \cdot BQ \cdot \cos \angle AQB \Rightarrow$$

$$AB = \sqrt{(601,6)^2 + (874,7)^2 - 2 \cdot 601,6 \cdot 874,7 \cdot \cos 59^\circ} \approx 764,8 \text{ m}$$



3.



Noem weer $\angle A = \alpha = 38^\circ$, $\angle B = \beta$ en $\angle C = \gamma$. We passen weer de sinusregel toe:

$$\frac{\sin \alpha}{BC} = \frac{\sin \beta}{AC} = \frac{\sin \gamma}{AB} \Rightarrow \frac{\sin 38^\circ}{5} = \frac{\sin \gamma}{8} \Rightarrow \sin \gamma = \frac{8 \cdot \sin 38^\circ}{5} \approx 0,98505$$

$$\Rightarrow \gamma \approx 80,08^\circ \text{ of } \gamma \approx 180^\circ - 80,08^\circ = 99,92^\circ$$

Hieruit volgt $\beta \approx 180^\circ - 80,08^\circ - 38^\circ = 61,92^\circ$ of $\beta \approx 180^\circ - 99,92^\circ - 38^\circ = 42,08^\circ$.

Voor AC geldt weer:

$$\frac{\sin \alpha}{BC} = \frac{\sin \beta}{AC}$$

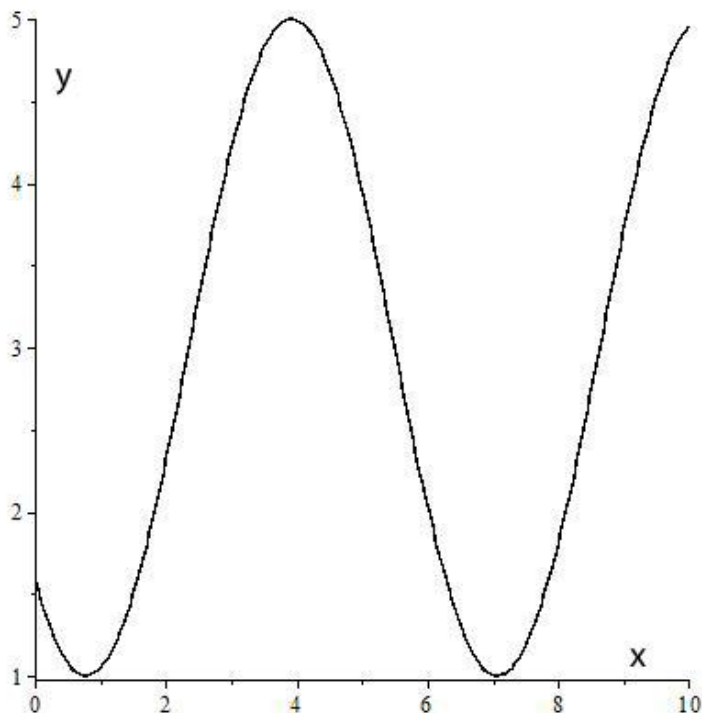
$$\Rightarrow AC \approx \frac{5 \sin(61,92^\circ)}{\sin 38^\circ} \approx 7,165 \text{ of } AC = \frac{BC \cdot \sin \beta}{\sin \alpha} \approx \frac{5 \sin(42,08^\circ)}{\sin 38^\circ} \approx 5,443$$

4.

a. De grafiek is getekend t.o.v. de "nullijn" $y = 1$: periode: $\frac{2\pi}{1} = 2\pi \approx 6,28$, amplitude: 2,

evenwichtslijn: $y = 3$, "beginpunt" 1: $(0; 3 - \sqrt{2}) \approx (0; 1,59)$, "beginpunt" 2:

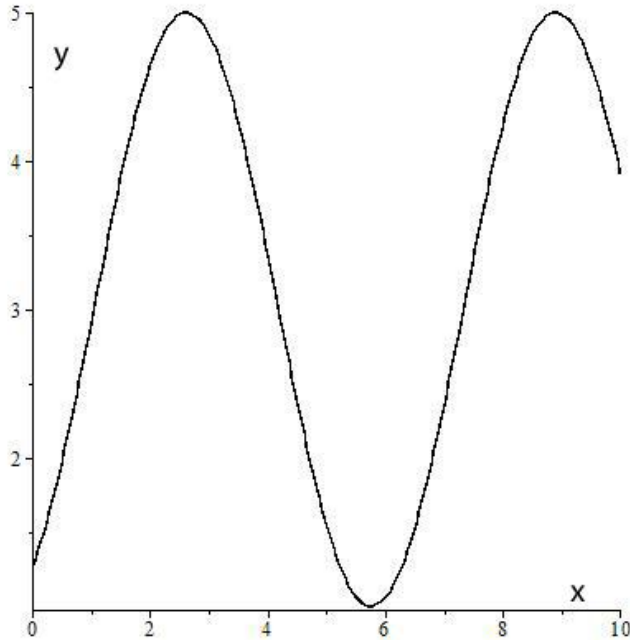
$$\left(\frac{1}{4}\pi; 1\right) \approx (0,785; 1)$$



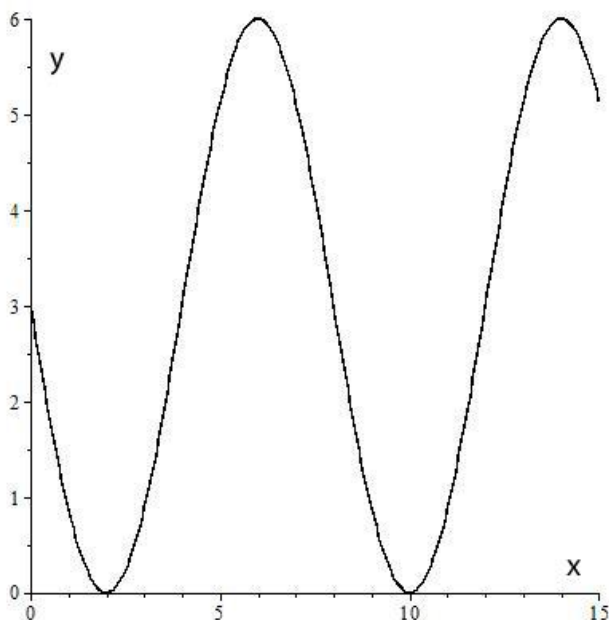


Toegepaste Wiskunde deel 1

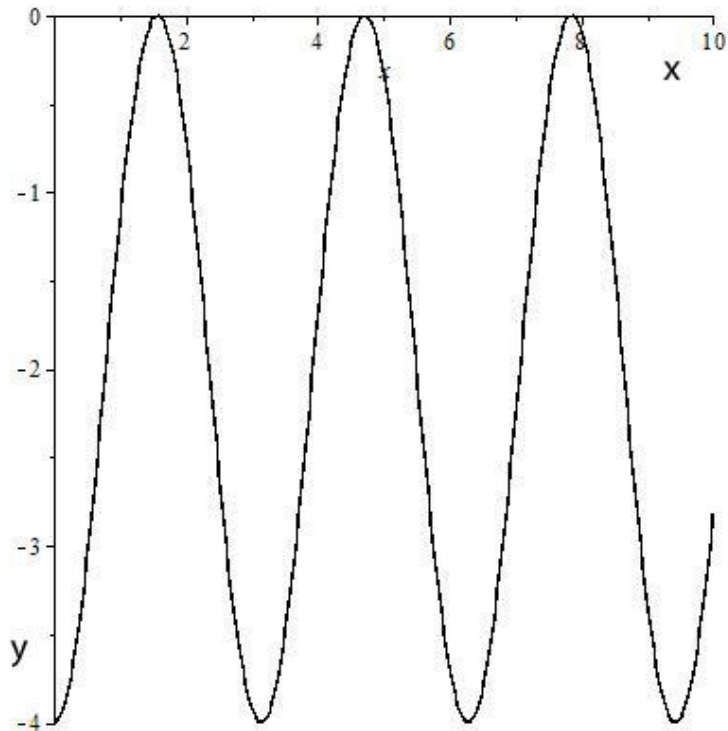
b. De grafiek is getekend t.o.v. de "nullijn" $y = 1$: periode: $\frac{2\pi}{1} = 2\pi \approx 6,28$, amplitude: 2, evenwichtslijn: $y = 3$, "beginpunt" 1: $(0; 3 - \sqrt{3}) \approx (0; 1,27)$, "beginpunt" 2: $(\frac{1}{3}\pi; 3) \approx (1,05; 3)$, "beginpunt" 3: $(\frac{4}{3}\pi; 3) \approx (4,19; 3)$



c. periode: $\frac{2\pi}{\frac{1}{4}\pi} = 8$, amplitude: 3, evenwichtslijn: $y = 3$, "beginpunt" 1: $(0, 3)$, "beginpunt" 2: $(2, 0)$



d. periode: $\frac{2\pi}{2} = \pi \approx 3,14$, amplitude: 2, evenwichtslijn: $y = -2$, "beginpunt" 1: $(0, -4)$, "beginpunt" 2: $(\frac{1}{4}\pi; -2) \approx (0,785; -2)$, "beginpunt" 3: $(\frac{1}{2}\pi; 0) \approx (1,57; 0)$



5.

a.

$$\begin{aligned}\frac{(\sin x)^3}{\cos x} + (\sin x)(\cos x) &= (\sin x) \left(\frac{(\sin x)^2}{\cos x} + \cos x \right) \\ &= (\sin x) \left(\frac{(\sin x)^2}{\cos x} + \frac{(\cos x)^2}{\cos x} \right) \\ &= (\sin x) \left(\frac{(\sin x)^2 + (\cos x)^2}{\cos x} \right) \\ &= \frac{\sin x}{\cos x} \\ &= \tan x\end{aligned}$$

b.

$$\begin{aligned}\frac{(\sin 2x)(\sin x)}{2 \cos x} + \cos 2x &= \frac{2 \sin x \cos x \sin x}{2 \cos x} + \cos 2x \\ &= (\sin x)^2 + (\cos x)^2 - (\sin x)^2 \\ &= (\cos x)^2\end{aligned}$$

6.

a.

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \Rightarrow \tan(2x) = \frac{2 \tan x}{1 - (\tan x)^2} \text{ (vul } y = x \text{ in).}$$



Toegepaste Wiskunde deel 1

b.

Stel $BC = y$. Uit de figuur volgt:

$$\tan(2x) = \frac{y}{6} \Rightarrow y = 6 \tan(2x) \text{ en } \tan x = \frac{y}{17} \Rightarrow y = 17 \tan x$$

$$\text{Dus } 6 \tan(2x) = 17 \tan x \Rightarrow 6 \frac{2 \tan x}{1 - (\tan x)^2} = 17 \tan x \Rightarrow \tan x = 0 \text{ of } 12 = 17 - 17(\tan x)^2.$$

De oplossing $\tan x = 0$ vervalt ($x \neq 0^\circ$).

$$\text{Blijft over: } (\tan x)^2 = \frac{5}{17} \Rightarrow \tan x = \sqrt{\frac{5}{17}} \approx 0,542 \Rightarrow x \approx \arctan(0,542) \approx 0,497 \text{ radialen.}$$

Conclusie: $x \approx 28,47^\circ$. Merk op dat de oplossing $\tan x = -\sqrt{\frac{5}{17}}$ vervalt ($x > 0$).

$$7. \sin \alpha = 0,4 \Rightarrow \cos \alpha = \pm \sqrt{1 - (0,4)^2} = \pm 0,3 \text{ (gebruik}$$

$$(\sin \alpha)^2 + (\cos \alpha)^2 = 1 \Rightarrow \cos \alpha = \pm \sqrt{1 - (\sin \alpha)^2}).$$

In het tweede kwadrant is $\cos \alpha$ negatief, dus $\cos \alpha = -0,3$.

$$\cos(2\alpha) = 1 - 2(\sin \alpha)^2 = 1 - 2 \cdot 0,16 = 0,68.$$

$$\sin(2\alpha) = 2 \sin \alpha \cos \alpha = 2 \cdot 0,4 \cdot (-0,3) = -0,24.$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{0,4}{-0,3} = -\frac{4}{3} \approx -1,333.$$

$$\tan(2\alpha) = \frac{\sin(2\alpha)}{\cos(2\alpha)} = \frac{-0,24}{0,68} \approx -0,353.$$

8. In alle hieronder gegeven oplossingen is k steeds een geheel getal.

a.

$$\sin(3x) = \frac{1}{2} \Rightarrow \sin(3x) = \sin\left(\frac{1}{6}\pi\right) \Rightarrow 3x = \frac{1}{6}\pi + k \cdot 2\pi \vee 3x = \pi - \frac{1}{6}\pi + k \cdot 2\pi = \frac{5}{6}\pi + k \cdot 2\pi$$

Hieruit volgt:

$$x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi \vee x = \frac{5}{18}\pi + k \cdot \frac{2}{3}\pi.$$

b.

$$3 \cos\left(x + \frac{1}{4}\pi\right) = 1 \Rightarrow \cos\left(x + \frac{1}{4}\pi\right) = \frac{1}{3} \Rightarrow \cos\left(x + \frac{1}{4}\pi\right) = \arccos\left(\frac{1}{3}\right) \approx 1,231$$

$$\Rightarrow x + \frac{1}{4}\pi \approx \pm 1,231 + k \cdot 2\pi$$

$$x \approx -\frac{1}{4}\pi + 1,231 + k \cdot 2\pi \approx -0,785 + 1,231 + k \cdot 2\pi \approx 0,446 + k \cdot 2\pi \text{ of}$$

$$x \approx -\frac{1}{4}\pi - 1,231 + k \cdot 2\pi \approx -0,785 - 1,231 + k \cdot 2\pi \approx -2,016 + k \cdot 2\pi$$

c.

$$2(\cos x)^2 - \cos x - 1 = 0 \Rightarrow 2p^2 - p - 1 = 0 \text{ (stel } p = \cos x) \Rightarrow p = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4}$$

$$p = -1 \Rightarrow \cos x = -1 = \cos \pi \Rightarrow x = \pm \pi + k \cdot 2\pi = \pi + k \cdot 2\pi \text{ of}$$

$$p = \frac{1}{2} = \cos\left(\frac{2}{3}\pi\right) \Rightarrow \cos x = \cos\left(\frac{2}{3}\pi\right) \Rightarrow x = \pm \frac{2}{3}\pi + k \cdot 2\pi$$



d.

$$\begin{aligned}\cos(3x - \frac{1}{3}\pi) &= -0,4 \approx \cos(1,982) \Rightarrow 3x - \frac{1}{3}\pi \approx \pm 1,982 + k \cdot 2\pi \\ 3x &\approx \frac{1}{3}\pi + 1,982 + k \cdot 2\pi \approx 1,047 + 1,982 + k \cdot 2\pi = 3,029 + k \cdot 2\pi \text{ of} \\ 3x &\approx \frac{1}{3}\pi - 1,982 + k \cdot 2\pi \approx 1,047 - 1,982 + k \cdot 2\pi = -0,935 + k \cdot 2\pi\end{aligned}$$

Dus

$$x \approx 1,009 + k \cdot \frac{2}{3}\pi \text{ of } x \approx -0,312 + k \cdot \frac{2}{3}\pi$$

e.

$$\tan(5x) = \sqrt{3} \Rightarrow \tan(5x) = \tan(\frac{1}{3}\pi) \Rightarrow 5x = \frac{1}{3}\pi + k \cdot \pi \Rightarrow x = \frac{1}{15}\pi + k \cdot \frac{1}{5}\pi$$

f.

$$\begin{aligned}(\sin x)^2 - 2(\cos x)^2 &= 0 \Rightarrow 1 - (\cos x)^2 - 2(\cos x)^2 = 0 \Rightarrow 3(\cos x)^2 = 1 \\ \Rightarrow \cos x &= \pm \sqrt{\frac{1}{3}} = \pm \frac{1}{\sqrt{3}} \approx \pm 0,577 \\ \cos x \approx 0,577 &\approx \cos(0,599) \Rightarrow x = \pm 0,599 + k \cdot 2\pi \text{ of} \\ \cos x \approx -0,577 &\approx \cos(2,186) \Rightarrow x = \pm 2,186 + k \cdot 2\pi\end{aligned}$$

g.

$$\sin(2x) = \cos(2x) \Rightarrow \frac{\sin(2x)}{\cos(2x)} = \tan(2x) = 1 = \tan(\frac{1}{4}\pi) \Rightarrow 2x = \frac{1}{4}\pi + k \cdot 2\pi \Rightarrow x = \frac{1}{8}\pi + k \cdot \pi$$

h.

$$\begin{aligned}\sin(2x - \frac{1}{3}) &= -0,6 \Rightarrow \sin(2x - \frac{1}{3}) \approx \sin(-0,644) \Rightarrow \\ 2x - \frac{1}{3} &\approx -0,644 + k \cdot 2\pi \Rightarrow 2x \approx 0,333 - 0,644 + k \cdot 2\pi \approx -0,311 + k \cdot 2\pi \Rightarrow x \approx -1,556 + k \cdot \pi \\ \text{of} \\ 2x - \frac{1}{3} &\approx \pi - (-0,644) + k \cdot 2\pi \Rightarrow 2x \approx 0,333 + 3,14 + 0,644 + k \cdot 2\pi \approx 4,119 + k \cdot 2\pi \Rightarrow x \approx 2,059 + k \cdot \pi\end{aligned}$$

9.

a.

$$\arcsin x = -\frac{1}{2}\pi \Rightarrow x = -1$$

b.

$$\arctan x = \frac{1}{3}\pi \Rightarrow x = \sqrt{3}$$

c.

$$\arccos x = \frac{1}{4}\pi \Rightarrow x = \frac{1}{2}\sqrt{2}$$

d.

$$\arcsin(2x - 1) = -\frac{1}{2}\pi \Rightarrow 2x - 1 = -1 \Rightarrow 2x = 0 \Rightarrow x = 0$$

e.

$$\arctan(3x + 1) = -0,4 \Rightarrow 3x + 1 = \tan(-0,4) \Rightarrow 3x + 1 \approx -0,423 \Rightarrow 3x \approx -0,523 \Rightarrow x \approx -0,174$$



f.

$$\arccos(2x+1) = \pi \Rightarrow 2x+1 = -1 \Rightarrow 2x = -2 \Rightarrow x = -1$$

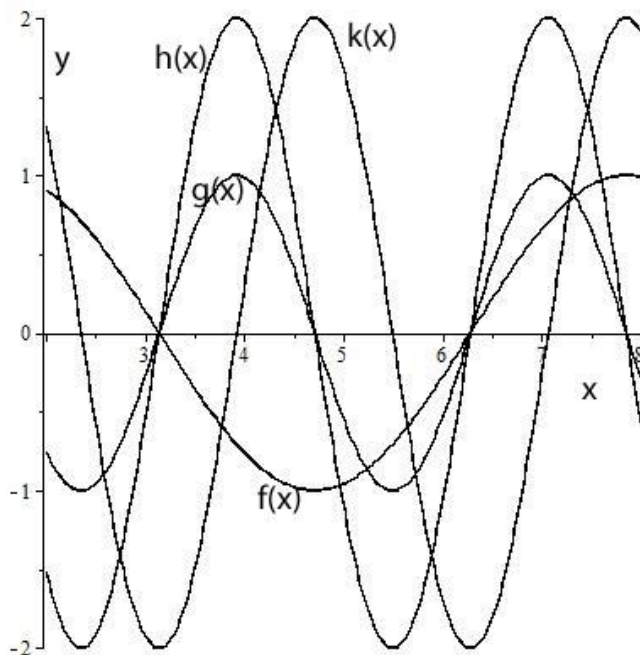
g.

$$\arcsin(-x) = 1 \Rightarrow -x = \frac{1}{2}\pi \Rightarrow x = -\frac{1}{2}\pi$$

h.

$$\arcsin(\sin x) = \frac{1}{2}\pi \Rightarrow x = \frac{1}{2}\pi$$

10.



11.

De drie hoeken bereken we met de cosinusregel:

$$BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos(\angle A) \Rightarrow 25 = 64 + 121 - 2 \cdot 8 \cdot 11 \cdot \cos(\angle A) \Rightarrow$$

$$\cos(\angle A) = \frac{25 - 185}{-176} = \frac{160}{176} \approx 0,9090 \Rightarrow \angle A \approx 0,4297 \text{ rad} \approx 24,62^\circ$$

$$AB^2 = AC^2 + BC^2 - 2 \cdot AC \cdot BC \cdot \cos(\angle C) \Rightarrow 64 = 121 + 25 - 2 \cdot 11 \cdot 5 \cdot \cos(\angle C) \Rightarrow$$

$$\cos(\angle C) = \frac{64 - 146}{-110} = \frac{82}{110} \approx 0,7455 \Rightarrow \angle C \approx 0,7296 \text{ rad} \approx 41,80^\circ$$

$$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos(\angle B) \Rightarrow 121 = 64 + 25 - 2 \cdot 8 \cdot 5 \cdot \cos(\angle B) \Rightarrow$$

$$\cos(\angle B) = \frac{121 - 89}{-80} = -\frac{32}{80} = -0,4000 \Rightarrow \angle B \approx 1,9823 \text{ rad} = 113,58^\circ$$

$$\text{Controle: } \angle A + \angle B + \angle C = 24,62^\circ + 41,80^\circ + 113,58^\circ = 180^\circ$$



12.

$$\cos \alpha = 0,3 \Rightarrow \sin \alpha = \pm \sqrt{1 - (0,3)^2} = \pm 0,4 \Rightarrow \tan \alpha = \pm \frac{0,4}{0,3} = \pm \frac{4}{3}$$

a. $\cos \alpha = 0,3 \Rightarrow \sin \alpha = 0,4 \Rightarrow \tan \alpha = \frac{4}{3}$

b. $\cos \alpha = 0,3 \Rightarrow \sin \alpha = -0,4 \Rightarrow \tan \alpha = -\frac{4}{3}$

c. $\cos \alpha = -0,3 \Rightarrow \sin \alpha = 0,4 \Rightarrow \tan \alpha = -\frac{4}{3}$

d. $\cos \alpha = -0,3 \Rightarrow \sin \alpha = -0,4 \Rightarrow \tan \alpha = \frac{4}{3}$