



Uitwerkingen extra opgaven hoofdstuk 2 Machten, wortels en logaritmen.

1. a $\frac{5^6}{5^2} = 5^{6-2} = 5^4$, dus antwoord B.

b $\frac{6^6}{2^6} = \left(\frac{6}{2}\right)^6 = 3^6$, dus antwoord D.

c $\frac{7^9}{7^{-3}} = 7^{9-(-3)} = 7^{12}$, dus antwoord D.

2. a $\frac{(-6)^6}{-2^6} = \frac{(-1)^6 \cdot 6^6}{-2^6} = -\frac{6^6}{2^6} = -\left(\frac{6}{2}\right)^6 = -3^6$, dus antwoord D.

b $3 - 2^4 = 3 - 16 = -13$, dus antwoord B.

c $3 \cdot (-2^2) = 3 \cdot (-4) = -12$, dus antwoord C.

3. a $\sqrt{10000} = \sqrt{10^4} = 10^2 = 100$

b $\sqrt{169} = \sqrt{13^2} = 13$

c $\sqrt{-4 + 20} = \sqrt{16} = 4$

d $\sqrt{-0,01}$ bestaat niet, geen enkel reëel getal heeft een negatief kwadraat.

4. a $\sqrt[3]{-27} = \sqrt[3]{(-3)^3} = -3$

b $\sqrt[4]{81} = \sqrt[4]{3^4} = 3$

c $\sqrt[4]{-16}$ bestaat niet, de vierde macht van een reëel getal is nooit negatief.

d $\sqrt[5]{-243} = \sqrt[5]{(-3)^5} = -3$

5. a $\sqrt{\frac{400}{81}} = \frac{\sqrt{400}}{\sqrt{81}} = \frac{20}{9} = 2\frac{2}{9}$

b $\sqrt{\frac{1}{36}} = \frac{\sqrt{1}}{\sqrt{36}} = \frac{1}{6}$

c $\sqrt[5]{-\frac{1}{32}} = \frac{\sqrt[5]{-1}}{\sqrt[5]{32}} = \frac{-1}{2} = -\frac{1}{2}$

d $\sqrt[4]{3\frac{13}{81}} = \sqrt[4]{\frac{256}{81}} = \frac{\sqrt[4]{256}}{\sqrt[4]{81}} = \frac{4}{3} = 1\frac{1}{3}$

6. a $\sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

b $1 - \sqrt{\frac{9}{25}} = 1 - \frac{3}{5} = \frac{2}{5}$

c $\sqrt{1} - \sqrt{\frac{9}{25}} = 1 - \frac{3}{5} = \frac{2}{5}$

d $\sqrt{1} + \sqrt{-\frac{9}{25}}$, bestaat niet, omdat $\sqrt{-\frac{9}{25}}$ niet bestaat.



Toegepaste Wiskunde inleiding

7. a $\sqrt[6]{4} \cdot \sqrt[3]{4} = \sqrt[6]{2^2} \cdot \sqrt[3]{2^2} = 2^{\frac{2}{6}} \cdot 2^{\frac{2}{3}} = 2^{\frac{2}{6} + \frac{2}{3}} = 2^1 = 2$
- b $\sqrt[3]{9} \cdot \sqrt[2]{27} = \sqrt[3]{3^2} \cdot \sqrt[2]{3^3} = 3^{\frac{2}{3}} \cdot 3^{\frac{3}{2}} = 3^{\frac{2}{3} + \frac{3}{2}} = 3^{\frac{4+9}{6}} = 3^{\frac{13}{6}} = 3^{2\frac{1}{6}}$
- c $\sqrt[5]{25} \cdot \sqrt[4]{5} = \sqrt[5]{5^2} \cdot \sqrt[4]{5^1} = 5^{\frac{2}{5}} \cdot 5^{\frac{1}{4}} = 5^{\frac{2}{5} + \frac{1}{4}} = 5^{\frac{8+5}{20}} = 5^{\frac{13}{20}}$
- d $\frac{5^3}{\sqrt[3]{25^2}} = \frac{5^3}{\sqrt[3]{(5^2)^2}} = \frac{5^3}{\sqrt[3]{5^4}} = \frac{5^3}{5^{\frac{4}{3}}} = 5^{3 - \frac{4}{3}} = 5^{\frac{9-4}{3}} = 5^{\frac{5}{3}} = 5^{1\frac{2}{3}}$
- e $\frac{3^4}{\sqrt[4]{27}} = \frac{3^4}{\sqrt[4]{3^3}} = \frac{3^4}{3^{\frac{3}{4}}} = 3^{4 - \frac{3}{4}} = 3^{\frac{16-3}{4}} = 3^{\frac{13}{4}} = 3^{3\frac{1}{4}}$
- f $\sqrt[6]{\frac{1}{25}} \cdot \frac{\sqrt[3]{25}}{5\sqrt{5}} = \sqrt[6]{5^{-2}} \cdot \frac{\sqrt[3]{5^2}}{5 \cdot 5^{\frac{1}{2}}} = 5^{-\frac{2}{6}} \cdot \frac{5^{\frac{2}{3}}}{5^{\frac{1}{2} + 1}} = 5^{-\frac{2}{6} + \frac{2}{3} - 1\frac{1}{2}} = 5^{-\frac{2}{6} + \frac{4}{6} - \frac{9}{6}} = 5^{-\frac{7}{6}} = 5^{-1\frac{1}{6}}$
8. a $12 \cdot 3^{\frac{2}{3}} = 2^2 \cdot 3 \cdot 3^{\frac{2}{3}} = 2^2 \cdot 3^{1 + \frac{2}{3}} = 2^2 \cdot 3^{\frac{5}{3}} = 2^2 \cdot \sqrt[3]{3^5}$
- b $3 \cdot 3^{\frac{1}{4}} = 3^{1 + \frac{1}{4}} = 3^{\frac{5}{4}} = 3^{\frac{9}{4}} = \sqrt[4]{3^9}$
- c $5^{-\frac{1}{2}} \cdot 10^{\frac{2}{3}} = 5^{-\frac{1}{2}} \cdot (2 \cdot 5)^{\frac{2}{3}} = 5^{-\frac{1}{2}} \cdot 2^{\frac{2}{3}} \cdot 5^{\frac{2}{3}} = 2^{\frac{8}{3}} \cdot 5^{-\frac{1}{2} + \frac{2}{3}} = 2^{\frac{8}{3}} \cdot 5^{-\frac{3}{6} + \frac{4}{6}} = 2^{\frac{8}{3}} \cdot 5^{\frac{1}{6}} = \sqrt[3]{2^8} \cdot \sqrt[6]{5^{13}}$
- d $6^2 \cdot 3^{-4} \cdot 2^{\frac{3}{2}} = (2 \cdot 3)^2 \cdot 3^{-4} \cdot 2^{\frac{3}{2}} = 2^2 \cdot 3^2 \cdot 3^{-4} \cdot 2^{\frac{3}{2}} = 2^{2 + \frac{3}{2}} \cdot 3^{2-4} = 2^{2\frac{4+3}{2}} \cdot 3^{-2} = \frac{2^{\frac{7}{2}}}{3^2} = \frac{\sqrt{2^7}}{3^2}$
9. a $\frac{3^{-4}}{5^{-3}} = 3^{-4} \cdot 5^3 = \frac{5^3}{3^4}$
- b $\frac{2^{-3}}{3^{-4}} = 2^{-3} \cdot 3^4 = \frac{3^4}{2^3}$
- c $\frac{2^{\frac{1}{4}} \cdot 10^{\frac{1}{2}}}{15^{\frac{2}{3}}} = \frac{2^{\frac{1}{4}} \cdot (2 \cdot 5)^{\frac{1}{2}}}{(3 \cdot 5)^{\frac{2}{3}}} = \frac{2^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot 5^{\frac{1}{2}}}{3^{\frac{2}{3}} \cdot 5^{\frac{2}{3}}} = 2^{\frac{1}{4} + \frac{1}{2}} \cdot 3^{-\frac{2}{3}} \cdot 5^{-\frac{1}{2} - \frac{2}{3}}$
 $= 2^{\frac{1}{4} + \frac{2}{4}} \cdot 3^{-\frac{8}{12}} \cdot 5^{-\frac{3}{6} - \frac{4}{6}} = 2^{\frac{3}{4}} \cdot 3^{-\frac{8}{12}} \cdot 5^{-\frac{7}{6}} = \frac{1}{2^{\frac{1}{4}} \cdot 3^{\frac{8}{12}} \cdot 5^{\frac{7}{6}}} = \frac{1}{\sqrt[4]{2} \cdot \sqrt[3]{3^8} \cdot \sqrt[6]{5^{19}}}$
- d $\frac{15}{\frac{5}{3}} \cdot 5^{-\frac{3}{4}} = 3 \cdot 5 \cdot 3^{-\frac{5}{3}} \cdot 5^{-\frac{3}{4}} = 3^{1 - \frac{5}{3}} \cdot 5^{-1 - \frac{3}{4}} = 3^{-\frac{2}{3}} \cdot 5^{-\frac{7}{4}} = \frac{5^{\frac{1}{4}}}{3^{\frac{2}{3}}} = \frac{\sqrt[4]{5}}{\sqrt[3]{3^2}}$
10. a ${}^3\log 9 = 2$, want $3^2 = 9$, of: ${}^3\log 9 = {}^3\log(3^2) = 2$.
- b ${}^3\log(-9)$ bestaat niet want geen enkele macht van 3 is gelijk aan -9 .
- c ${}^3\log 0$ bestaat niet want geen enkele macht van 3 is geheel aan 0.
- d $\frac{1}{5}\log 5 = -1$, want $5 = 5^1 = \left(\frac{1}{5}\right)^{-1}$, of: $\frac{1}{5}\log 5 = \frac{1}{5}\log\left(\frac{1}{5}\right)^{-1} = -1$.
- e $\frac{1}{5}\log(5^{-2}) = 2$, want $5^{-2} = (5^{-1})^2 = \left(\frac{1}{5}\right)^2$, of: $\frac{1}{5}\log(5^{-2}) = \frac{1}{5}\log\left(\left(\frac{1}{5}\right)^2\right) = 2$.



f $\frac{1}{5} \log\left(\frac{1}{\sqrt{5}}\right) = \frac{1}{2}$, want $\frac{1}{\sqrt{5}} = \frac{1^{\frac{1}{2}}}{5^{\frac{1}{2}}} = \left(\frac{1}{5}\right)^{\frac{1}{2}}$, of: $\frac{1}{5} \log\left(\frac{1}{\sqrt{5}}\right) = \frac{1}{5} \log\left(\sqrt{\frac{1}{5}}\right) = \frac{1}{5} \log\left(\left(\frac{1}{5}\right)^{\frac{1}{2}}\right) = \frac{1}{2}$.

11. a ${}^2 \log(0,125) = -3$, want $0,125 = \frac{125}{1000} = \frac{1}{8} = 2^{-3}$, of:

$${}^2 \log(0,125) = {}^2 \log\left(\frac{125}{1000}\right) = {}^2 \log\left(\frac{1}{8}\right) = {}^2 \log(2^{-3}) = -3$$

b ${}^3 \log(-27)$, bestaat niet want geen enkele macht van 3 is gelijk aan -27 .

c ${}^7 \log\left(\frac{\sqrt{7}}{49}\right) = -1\frac{1}{2}$, want $\frac{\sqrt{7}}{49} = \frac{7^{\frac{1}{2}}}{7^2} = 7^{\frac{1}{2}-2} = 7^{-1\frac{1}{2}}$, of:

$${}^7 \log\left(\frac{\sqrt{7}}{49}\right) = {}^7 \log\left(\frac{7^{\frac{1}{2}}}{7^2}\right) = {}^7 \log(7^{\frac{1}{2}-2}) = {}^7 \log(7^{-1\frac{1}{2}}) = -1\frac{1}{2}$$

d $\frac{1}{5} \log 5 = -1$, want $5 = \left(\frac{1}{5}\right)^{-1}$, of: $\frac{1}{5} \log 5 = \frac{1}{5} \log\left(\left(\frac{1}{5}\right)^{-1}\right) = -1$.

e $\frac{1}{3} \log 1 = 0$, want $1 = \left(\frac{1}{3}\right)^0$, of: $\frac{1}{3} \log 1 = \frac{1}{3} \log\left(\frac{1}{3}\right)^0 = 0$.

f $\frac{1}{5} \log\left(\frac{1}{25\sqrt{5}}\right) = -2\frac{1}{2}$, want $\frac{1}{25\sqrt{5}} = \frac{1}{5^2 \cdot 5^{\frac{1}{2}}} = \frac{1}{5^{2\frac{1}{2}}}$, of:

$$\frac{1}{5} \log\left(\frac{1}{25\sqrt{5}}\right) = \frac{1}{5} \log\left(\frac{1}{5^2 \cdot 5^{\frac{1}{2}}}\right) = \frac{1}{5} \log\left(\frac{1}{5^{2\frac{1}{2}}}\right) = \frac{1}{5} \log(5^{-2\frac{1}{2}}) = -2\frac{1}{2}$$

12. a ${}^5 \log 2 = \frac{\log 2}{\log 5} \approx 0,4307$

b ${}^3 \log(-25)$, bestaat niet, want geen enkele macht van 3 is gelijk aan -25

c ${}^5 \log(11,2) = \frac{\log 11,2}{\log 5} \approx 1,5011$

d $\frac{1}{3} \log 5 = \frac{\log 5}{\log\left(\frac{1}{3}\right)} = \frac{\log 5}{\log(3^{-1})} = \frac{\log 5}{-\log(3)} \approx -1,4650$ of directer: $\frac{1}{3} \log 5 = \frac{\log 5}{\log\left(\frac{1}{3}\right)} \approx -1,4650$

e $\log\left(\frac{1}{17,2}\right) = \log\left((17,2)^{-1}\right) = -\log(17,2) \approx -1,2355$ of direct $\log\left(\frac{1}{17,2}\right) \approx -1,2355$

f $\sqrt{2} \log 5 = \frac{\log 5}{\log(\sqrt{2})} = \frac{\log 5}{\log\left(2^{\frac{1}{2}}\right)} = \frac{\log 5}{\frac{1}{2} \cdot \log(2)} = 2 \cdot \frac{\log 5}{\log 2} \approx 4,6439$

of directer: $\sqrt{2} \log 5 = \frac{\log 5}{\log(\sqrt{2})} \approx 4,6439$

13. a $\log 50 = \log 2 + \log 25$; *waar*, want $\log 2 + \log 25 = \log(2 \cdot 25) = \log 50$

b $\log 50 = \log 20 + \log 30$; *niet waar*, want $\log 20 + \log 30 = \log(20 \cdot 30) = \log(600) \neq \log 50$.

c $\log 50 = 2 \cdot \log 25$; *niet waar*, want $2 \cdot \log 25 = \log(25^2) = \log 625 \neq \log 50$.

d $\log 50 = 1 + \log 5$; *waar*, want $\log 50 = \log(10 \cdot 5) = \log 10 + \log 5 = 1 + \log 5$.



Toegepaste Wiskunde inleiding

- 14.**
- a $\log 70 = \log 2 + \log 35$; *waar*, want $\log 2 + \log 35 = \log(2 \cdot 35) = \log 70$
 - b $\log 70 = \log 10 + \log 60$; *niet waar*, want $\log 10 + \log 60 = \log(10 \cdot 60) = \log(600) \neq \log 70$.
 - c $\log 70 = 10 \cdot \log 7$; *niet waar*, want $10 \cdot \log 7 = \log(7^{10}) \neq \log 70$.
 - d $\log 70 = 1 + \log 7$; *waar*, want $\log 70 = \log(10 \cdot 7) = \log 10 + \log 7 = 1 + \log 7$.