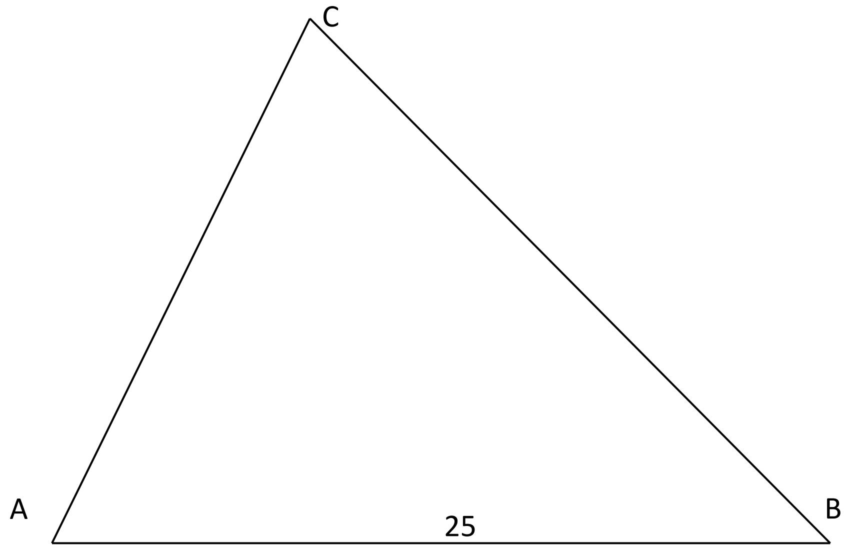
Uitwerkingen extra opgaven hoofdstuk 3 Goniometrie

1.



Noem $\angle A = \alpha$, $\angle B = \beta$ en $\angle C = \gamma$.

$\alpha = 74°, \beta = 46° \Rightarrow \gamma = 180° - 74° - 46° = 60°$.

Met de sinusregel berekenen we AC en BC:

$$\frac{\sin\alpha}{BC} = \frac{\sin\beta}{AC} = \frac{\sin\gamma}{AB} \Rightarrow \frac{\sin 74°}{BC} = \frac{\sin 46°}{AC} = \frac{\sin 60°}{25} \approx 0,03464$$.

Hieruit volgt: $BC \approx \frac{\sin 74°}{0,03464} \approx 27,75$ en $AC \approx \frac{\sin 46°}{0,03464} \approx 20,77$

2.

Uit de gegevens volgt: $\angle BQP = 59° + 33° = 92°$ en $\angle APQ = 70° + 55° = 125°$.

We passen nu de sinusregel toe in driehoek BPQ om BQ te bepalen en we passen de sinusregel toe in driehoek APQ om AQ te bepalen:

$$\frac{\sin 55°}{BQ} = \frac{\sin(\angle PBQ)}{PQ} = \frac{\sin(180° - 55° - 92°)}{PQ} \Rightarrow BQ = PQ \cdot \frac{\sin 55°}{\sin 33°} \approx 400 \cdot \frac{0,81915}{0,54464} \approx 601,6$$

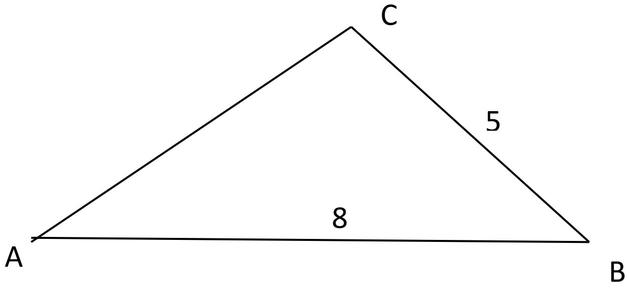
$$\frac{\sin 125°}{AQ} = \frac{\sin(\angle PAQ)}{PQ} = \frac{\sin(180° - 125° - 33°)}{PQ} \Rightarrow AQ = PQ \cdot \frac{\sin 125°}{\sin 22°} \approx 400 \cdot \frac{0,81915}{0,37461} \approx 874,7$$

Met de cosinusregel bepalen we nu AB:

$$AB^2 = AQ^2 + BQ^2 - 2AQ \cdot BQ \cdot \cos\angle AQB \Rightarrow$$

$$AB = \sqrt{(601,6)^2 + (874,7)^2 - 2 \cdot 601,6 \cdot 874,7 \cdot \cos 59°} \approx 764,8 \text{ m}$$

3.



Noem weer $\angle A = \alpha = 38°$, $\angle B = \beta$ en $\angle C = \gamma$. We passen weer de sinusregel toe:

$$\frac{\sin\alpha}{BC} = \frac{\sin\beta}{AC} = \frac{\sin\gamma}{AB} \Rightarrow \frac{\sin 38°}{5} = \frac{\sin\gamma}{8} \Rightarrow \sin\gamma = \frac{8\cdot\sin 38°}{5} \approx 0{,}98505 .$$

$\Rightarrow \gamma \approx 80{,}08°$ of $\gamma \approx 180° - 80{,}08° = 99{,}92°$

Hieruit volgt $\beta \approx 180° - 80{,}08° - 38° = 61{,}92°$ of $\beta \approx 180° - 99{,}92° - 38° = 42{,}08°$.

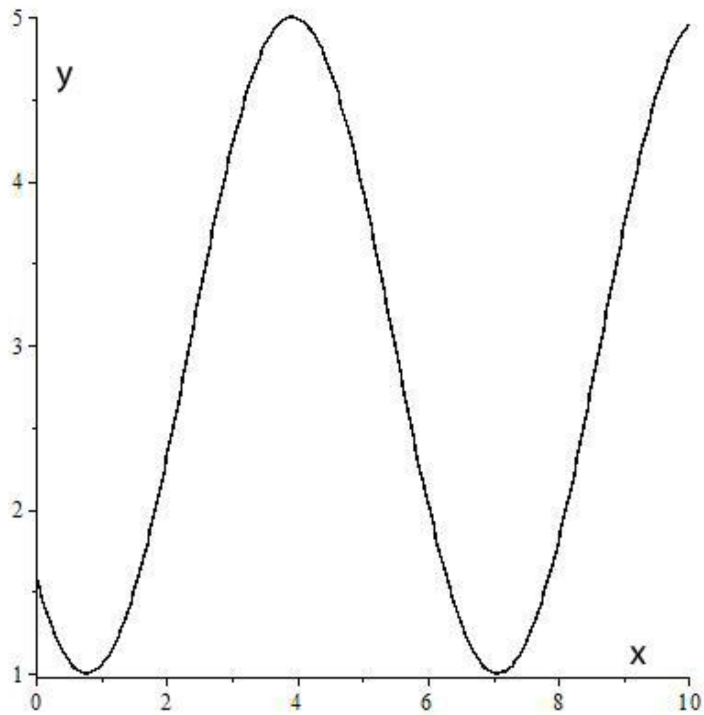
Voor AC geldt weer:

$$\frac{\sin\alpha}{BC} = \frac{\sin\beta}{AC}$$

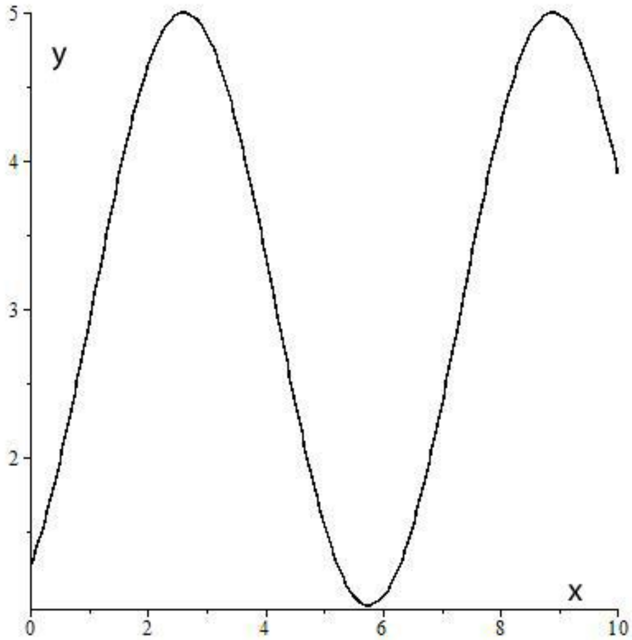
$$\Rightarrow AC \approx \frac{5\sin(61{,}92°)}{\sin 38°} \approx 7{,}165 \text{ of } AC = \frac{BC\cdot\sin\beta}{\sin\alpha} \approx \frac{5\sin(42{,}08°)}{\sin 38°} \approx 5{,}443$$

4.

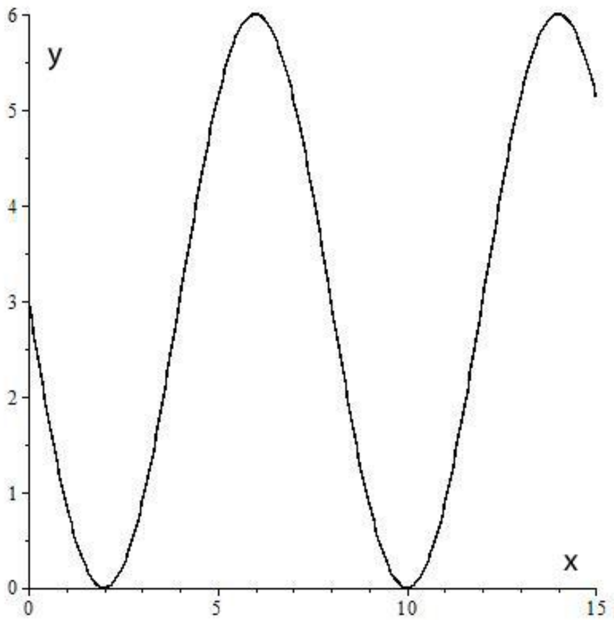
a.De grafiek is getekend t.o.v.de “nullijn” $y = 1$: periode: $\frac{2\pi}{1} = 2\pi \approx 6{,}28$, amplitude: 2, evenwichtslijn: $y = 3$, “beginpunt” 1: $\left(0;3-\sqrt{2}\right) \approx (0;1{,}59)$, “beginpunt” 2: $\left(\frac{1}{4}\pi;1\right) \approx (0{,}785;1)$



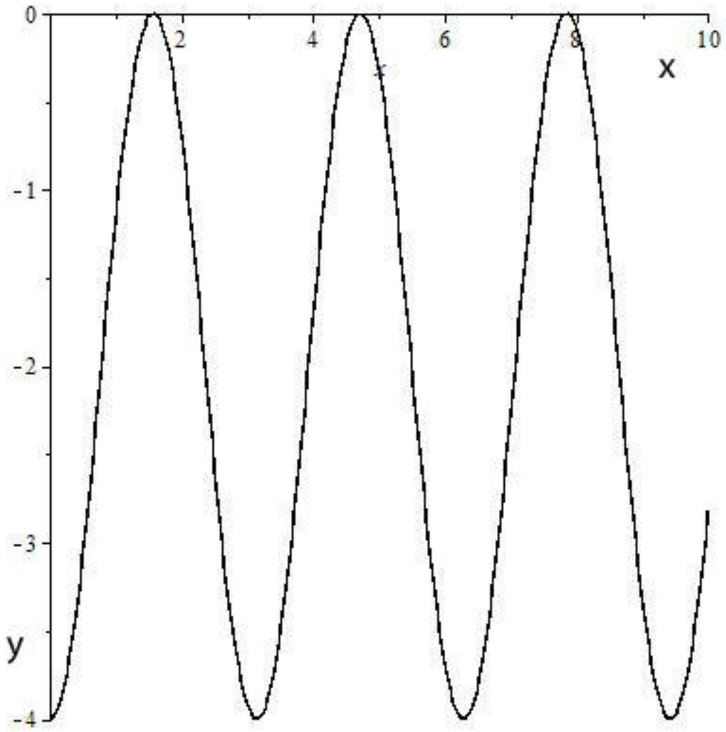
b. De grafiek is getekend t.o.v. de "nullijn" $y=1$: periode: $\frac{2\pi}{1}=2\pi\approx 6,28$, amplitude: 2, evenwichtslijn: $y=3$, "beginpunt" 1: $\left(0;3-\sqrt{3}\right)\approx(0;1,27)$, "beginpunt" 2: $\left(\frac{1}{3}\pi;3\right)\approx(1,05;3)$, "beginpunt" 3: $\left(\frac{4}{3}\pi;3\right)\approx(4,19;3)$



c. periode: $\frac{2\pi}{\frac{1}{4}\pi}=8$, amplitude: 3, evenwichtslijn: $y=3$, "beginpunt" 1: $(0,3)$, "beginpunt" 2: $(2,0)$



d. periode: $\frac{2\pi}{2}=\pi\approx 3,14$, amplitude: 2, evenwichtslijn: $y=-2$, "beginpunt" 1: $(0,-4)$, "beginpunt" 2: $\left(\frac{1}{4}\pi;-2\right)\approx(0,785;-2)$, "beginpunt" 3: $\left(\frac{1}{2}\pi;0\right)\approx(1,57;0)$



5.

a.

$$\frac{(\sin x)^3}{\cos x}+(\sin x)(\cos x)=(\sin x)\left(\frac{(\sin x)^2}{\cos x}+\cos x\right)$$
$$=(\sin x)\left(\frac{(\sin x)^2}{\cos x}+\frac{(\cos x)^2}{\cos x}\right)$$
$$=(\sin x)\left(\frac{(\sin x)^2+(\cos x)^2}{\cos x}\right)$$
$$=\frac{\sin x}{\cos x}$$
$$=\tan x$$

b.

$$\frac{(\sin 2x)(\sin x)}{2\cos x}+\cos 2x=\frac{2\sin x\cos x\sin x}{2\cos x}+\cos 2x$$
$$=(\sin x)^2+(\cos x)^2-(\sin x)^2$$
$$=(\cos x)^2$$

6.

a.

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}\Rightarrow \tan(2x)=\frac{2\tan x}{1-(\tan x)^2}$$ (vul $y=x$ in).

b.

Stel $\mathrm{BC}=y$. Uit de figuur volgt:

$\tan(2x)=\frac{y}{6}\Rightarrow y=6\tan(2x)$ en $\tan x=\frac{y}{17}\Rightarrow y=17\tan x$

Dus $6\tan(2x)=17\tan x\Rightarrow 6\frac{2\tan x}{1-(\tan x)^2}=17\tan x\Rightarrow \tan x=0$ of $12=17-17(\tan x)^2$.

De oplossing $\tan x=0$ vervalt ($x\neq 0°$).

Blijft over: $(\tan x)^2=\frac{5}{17}\Rightarrow \tan x=\sqrt{\frac{5}{17}}\approx 0{,}542\Rightarrow x\approx \arctan(0{,}542)\approx 0{,}497$ radialen.

Conclusie: $x\approx 28{,}47°$. Merk op dat de oplossing $\tan x=-\sqrt{\frac{5}{17}}$ vervalt ($x>0$).

7. $\sin\alpha=0{,}4\Rightarrow \cos\alpha=\pm\sqrt{1-(0{,}4)^2}=\pm 0{,}3$ (gebruik $(\sin\alpha)^2+(\cos\alpha)^2=1\Rightarrow \cos\alpha=\pm\sqrt{1-(\sin\alpha)^2}$).

In het tweede kwadrant is $\cos\alpha$ negatief, dus $\cos\alpha=-0{,}3$.

$\cos(2\alpha)=1-2(\sin\alpha)^2=1-2\cdot 0{,}16=0{,}68$.

$\sin(2\alpha)=2\sin\alpha\cos\alpha=2\cdot 0{,}4\cdot(-0{,}3)=-0{,}24$.

$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{0{,}4}{-0{,}3}=-\frac{4}{3}\approx -1{,}333$.

$\tan(2\alpha)=\frac{\sin(2\alpha)}{\cos(2\alpha)}=\frac{0{,}68}{-0{,}24}\approx -2{,}833$.

8.In alle hieronder gegeven oplossingen is k steeds een geheel getal.

a.

$\sin(3x)=\frac{1}{2}\Rightarrow \sin(3x)=\sin\left(\frac{1}{6}\pi\right)\Rightarrow 3x=\frac{1}{6}\pi+k\cdot 2\pi \vee 3x=\pi-\frac{1}{6}\pi+k\cdot 2\pi=\frac{5}{6}\pi+k\cdot 2\pi$

Hieruit volgt:

$x=\frac{1}{18}\pi+k\cdot\frac{2}{3}\pi\vee x=\frac{5}{18}\pi+k\cdot\frac{2}{3}\pi$.

b.

$3\cos(x+\frac{1}{4}\pi)=1\Rightarrow \cos(x+\frac{1}{4}\pi)=\frac{1}{3}\Rightarrow \cos(x+\frac{1}{4}\pi)=\arccos(\frac{1}{3})\approx 1{,}231$

$\Rightarrow x+\frac{1}{4}\pi\approx \pm 1{,}231+k\cdot 2\pi$

$x\approx -\frac{1}{4}\pi+1{,}231+k\cdot 2\pi\approx -0{,}785+1{,}231+k\cdot 2\pi\approx 0{,}446+k\cdot 2\pi$ of

$x\approx -\frac{1}{4}\pi-1{,}231+k\cdot 2\pi\approx -0{,}785-1{,}231+k\cdot 2\pi\approx -2{,}016+k\cdot 2\pi$

c.

$2(\cos x)^2-\cos x-1=0\Rightarrow 2p^2-p-1=0$ (stel $p=\cos x$) $\Rightarrow p=\frac{1\pm\sqrt{1+8}}{4}=\frac{1\pm 3}{4}$

$p=-1\Rightarrow \cos x=-1=\cos\pi\Rightarrow x=\pm\pi+k\cdot 2\pi=\pi+k\cdot 2\pi$ of

$p=-\frac{1}{2}=\cos(\frac{2}{3}\pi)\Rightarrow \cos x=\cos(\frac{2}{3}\pi)\Rightarrow x=\pm\frac{2}{3}\pi+k\cdot 2\pi$

d.

$\cos(3x-\frac{1}{3}\pi)=-0,4\approx\cos(1,982)\Rightarrow 3x-\frac{1}{3}\pi\approx\pm1,982+k\cdot2\pi$

$3x\approx\frac{1}{3}\pi+1,982+k\cdot2\pi\approx1,047+1,982+k\cdot2\pi=3,029+k\cdot2\pi$ of

$3x\approx\frac{1}{3}\pi-1,982+k\cdot2\pi\approx1,047-1,982+k\cdot2\pi=-0,935+k\cdot2\pi$

Dus

$x\approx1,009+k\cdot\frac{2}{3}\pi$ of $x\approx-0,312+k\cdot\frac{2}{3}\pi$

e.

$\tan(5x)=\sqrt{3}\Rightarrow\tan(5x)=\tan\left(\frac{1}{3}\pi\right)\Rightarrow 5x=\frac{1}{3}\pi+k\cdot\pi\Rightarrow x=\frac{1}{15}\pi+k\cdot\frac{1}{5}\pi$

f.

$(\sin x)^2-2(\cos x)^2=0\Rightarrow1-(\cos x)^2-2(\cos x)^2=0\Rightarrow3(\cos x)^2=1$

$\Rightarrow\cos x=\pm\sqrt{\frac{1}{3}}=\pm\frac{1}{3}\sqrt{3}\approx\pm0,577$

$\cos x\approx0,577\approx\cos(0,599)\Rightarrow x=\pm0,599+k\cdot2\pi$ of

$\cos x\approx-0,577\approx\cos(2,186)\Rightarrow x=\pm2,186+k\cdot2\pi$

g.

$\sin(2x)=\cos(2x)\Rightarrow\frac{\sin(2x)}{\cos(2x)}=\tan(2x)=1=\tan(\frac{1}{4}\pi)\Rightarrow 2x=\frac{1}{4}\pi+k\cdot2\pi\Rightarrow x=\frac{1}{8}\pi+k\cdot\pi$

h.

$\sin(2x-\frac{1}{3})=-0,6\Rightarrow\sin(2x-\frac{1}{3})\approx\sin(-0,644)\Rightarrow$

$2x-\frac{1}{3}\approx-0,644+k\cdot2\pi\Rightarrow 2x\approx0,333-0,644+k\cdot2\pi\approx-0,311+k\cdot2\pi\Rightarrow x\approx-1,556+k\cdot\pi$

of

$2x-\frac{1}{3}\approx\pi-(-0,644)+k\cdot2\pi\Rightarrow 2x\approx0,333+3,14+0,644+k\cdot2\pi\approx4,119+k\cdot2\pi\Rightarrow x\approx2,059+k\cdot\pi$

9.

a.

$\arcsin x=-\frac{1}{2}\pi\Rightarrow x=-1$

b.

$\arctan x=\frac{1}{3}\pi\Rightarrow x=\sqrt{3}$

c.

$\arccos x=\frac{1}{4}\pi\Rightarrow x=\frac{1}{2}\sqrt{2}$

d.

$\arcsin(2x-1)=-\frac{1}{2}\pi\Rightarrow 2x-1=-1\Rightarrow 2x=0\Rightarrow x=0$

e.

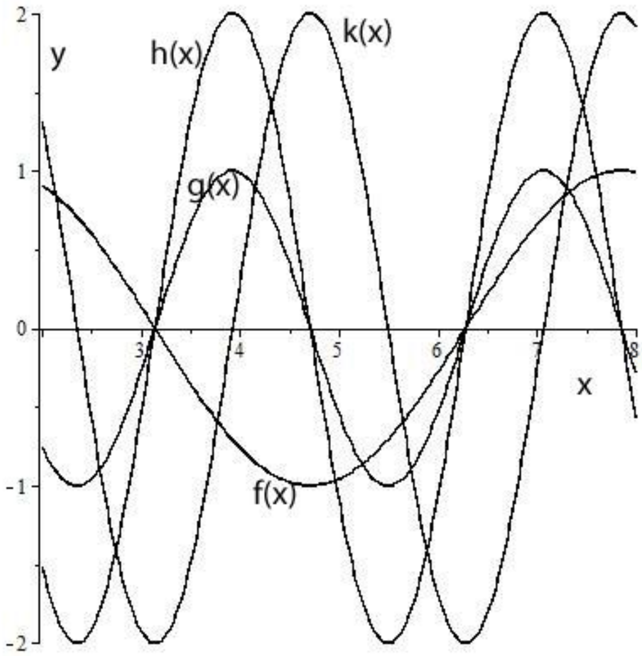
$\arctan(3x+1)=-0,4\Rightarrow 3x+1=\tan(-0,4)\Rightarrow 3x+1\approx-0,423\Rightarrow 3x\approx-0,523\Rightarrow x\approx-0,174$

f.
$$\arccos(2x+1)=\pi \Rightarrow 2x+1=-1 \Rightarrow 2x=-2 \Rightarrow x=-1$$

g.
$$\arcsin(-x)=1 \Rightarrow -x=\tfrac{1}{2}\pi \Rightarrow x=-\tfrac{1}{2}\pi$$

h.
$$\arcsin(\sin x)=\tfrac{1}{2}\pi \Rightarrow x=\tfrac{1}{2}\pi$$

10.



11.
De drie hoeken bereken we met de cosinusregel:

$$BC^2 = AB^2 + AC^2 - 2\cdot AB\cdot AC\cdot \cos(\angle A) \Rightarrow 25 = 64+121-2\cdot 8\cdot 11\cdot \cos(\angle A) \Rightarrow$$

$$\cos(\angle A)=\frac{25-185}{-176}=\frac{160}{176}\approx 0,9090 \Rightarrow \angle A \approx 0,4297 \text{ rad} \approx 24,62°$$

$$AB^2 = AC^2 + BC^2 - 2\cdot AC\cdot BC\cdot \cos(\angle C) \Rightarrow 64 = 121+25-2\cdot 11\cdot 5\cdot \cos(\angle C) \Rightarrow$$

$$\cos(\angle C)=\frac{64-146}{-110}=\frac{82}{110}\approx 0,7455 \Rightarrow \angle C \approx 0,7296 \text{ rad} \approx 41,80°$$

$$AC^2 = AB^2 + BC^2 - 2\cdot AB\cdot BC\cdot \cos(\angle B) \Rightarrow 121 = 64+25-2\cdot 8\cdot 5\cdot \cos(\angle B) \Rightarrow$$

$$\cos(\angle B)=\frac{121-89}{-80}=-\frac{32}{80}=-0,4000 \Rightarrow \angle B \approx 1,9823 \text{ rad} = 113,58°$$

Controle: $\angle A+\angle B+\angle C = 24,62°+41,80°+113,58° = 180°$

12.

$$\cos\alpha = 0,3 \Rightarrow \sin\alpha = \pm\sqrt{1-(0,3)^2} = \pm 0,4 \Rightarrow \tan\alpha = \pm\frac{0,4}{0,3} = \pm\tfrac{4}{3}$$

a. $\cos\alpha = 0,3 \Rightarrow \sin\alpha = 0,4 \Rightarrow \tan\alpha = \tfrac{4}{3}$

b. $\cos\alpha = 0,3 \Rightarrow \sin\alpha = -0,4 \Rightarrow \tan\alpha = -\tfrac{4}{3}$

c. $\cos\alpha = -0,3 \Rightarrow \sin\alpha = 0,4 \Rightarrow \tan\alpha = -\tfrac{4}{3}$

d. $\cos\alpha = -0,3 \Rightarrow \sin\alpha = -0,4 \Rightarrow \tan\alpha = \tfrac{4}{3}$