

drs. J.H. Blankespoor
drs. C. de Joode
ir. A. Sluijter

Toegepaste Wiskunde voor het hoger beroepsonderwijs

Deel 1

Zesde, herziene druk

Uitwerking herhalingsopgaven hoofdstuk 6 Integraalrekening

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Uitwerking herhalingsopgaven hoofdstuk 6, paragraaf 6.8

Opgave 1

a

$$\begin{aligned}\int 3x\sqrt{1-2x^2} \, dx &= -\frac{3}{4} \int \sqrt{1-2x^2} (1-2x^2)' \, dx \\ &= -\frac{3}{4} \int (1-2x^2)^{\frac{1}{2}} d(1-2x^2) \\ &= -\frac{3}{4} \frac{2}{3} (1-2x^2)^{\frac{3}{2}} + C \\ &= -\frac{1}{2} (1-2x^2) \sqrt{1-2x^2} + C\end{aligned}$$

b

$$\begin{aligned}\int \frac{8t^2}{(t^3+2)^3} \, dt &= \frac{8}{3} \int (t^3+2)^{-3} (t^3+2)' \, dt \\ &= \frac{8}{3} \int (t^3+2)^{-3} d(t^3+2) \\ &= \frac{8}{3} \left(-\frac{1}{2}\right) (t^3+2)^{-2} + C \\ &= -\frac{4}{3(t^3+2)^2} + C\end{aligned}$$

c

$$\int \frac{1}{5x^2+16} \, dx = \int \frac{1}{(x\sqrt{5})^2 + 4^2} \, dx$$

Stel $t = x\sqrt{5}$, dan $\frac{dt}{dx} = \sqrt{5}$, zodat $dx = \frac{1}{\sqrt{5}} dt$.

Er volgt:

$$\begin{aligned}\int \frac{1}{5x^2+16} \, dx &= \int \frac{1}{(x\sqrt{5})^2 + 4^2} \, dx \\ &= \frac{1}{\sqrt{5}} \int \frac{1}{t^2 + 4^2} \, dt \\ &= \frac{1}{4\sqrt{5}} \arctan\left(\frac{t}{4}\right) + C \\ &= \frac{1}{20} \sqrt{5} \arctan\left(\frac{x\sqrt{5}}{4}\right) + C\end{aligned}$$

d

$$\begin{aligned}\int_0^2 4pe^{3p^2-1} \, dp &= \frac{4}{6} \int_0^2 e^{3p^2-1} (3p^2-1)' \, dp \\ &= \frac{2}{3} \int_{p=0}^2 e^{3p^2-1} d(3p^2-1) \\ &= \frac{2}{3} \left[e^{3p^2-1} \right]_{p=0}^2 = \frac{2}{3} (e^{11} - e^{-1})\end{aligned}$$

e

$$\begin{aligned}\int v \sin(3v^2 - 5) dv &= \frac{1}{6} \int \sin(3v^2 - 5) d(3v^2 - 5) \\ &= -\frac{1}{6} \cos(3v^2 - 5) + C\end{aligned}$$

f

$$\begin{aligned}\int \frac{4}{x \ln^2 x} dx &= 4 \int (\ln x)^{-2} d \ln x \\ &= -\frac{4}{\ln x} + C\end{aligned}$$

g

$$\int \frac{1}{\sqrt{4-x^2}} dx = \int \frac{1}{\sqrt{2^2-x^2}} dx = \arcsin\left(\frac{1}{2}x\right) + C$$

h

$$\begin{aligned}\int \frac{x}{\sqrt{4-x^2}} dx &= -\frac{1}{2} \int (4-x^2)^{-\frac{1}{2}} d(4-x^2) \\ &= -\frac{1}{2}(-2)(4-x^2)^{-\frac{1}{2}} + C \\ &= -\sqrt{4-x^2} + C\end{aligned}$$

Opgave 2

a

$$\begin{aligned}\int_0^2 \frac{1}{x^2 - x + 1} dx &= \int_0^2 \frac{1}{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}} dx \\ &= \int_{x=0}^2 \frac{1}{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} d\left(x - \frac{1}{2}\right) \\ &= \frac{2}{\sqrt{3}} \left[\arctan\left(\frac{2x-1}{\sqrt{3}}\right) \right]_{x=0}^2 \\ &= \frac{2}{\sqrt{3}} \left(\arctan\left(\frac{3}{\sqrt{3}}\right) - \arctan\left(-\frac{1}{\sqrt{3}}\right) \right) \\ &= \frac{2}{3} \sqrt{3} \left(\frac{1}{3} \pi - \left(-\frac{1}{6} \pi\right) \right) = \frac{2}{3} \sqrt{3} \cdot \frac{1}{2} \pi = \frac{1}{3} \pi \sqrt{3}\end{aligned}$$

b

$$\begin{aligned}\int_0^{\ln 3} e^t \sqrt{e^t + 1} dt &= \int_{t=0}^{\ln 3} (e^t + 1)^{\frac{1}{2}} d(e^t + 1) \\ &= \left[\frac{2}{3} (e^t + 1)^{\frac{3}{2}} \right]_{t=0}^{\ln 3} \\ &= \frac{2}{3} \cdot 4^{\frac{3}{2}} - \frac{2}{3} \cdot 2^{\frac{3}{2}} = \frac{16}{3} - \frac{4}{3} \sqrt{2}\end{aligned}$$

c

$$\begin{aligned}\int \frac{7x}{5x^2+16} dx &= \frac{7}{10} \int \frac{(5x^2+16)'}{5x^2+16} dx \\ &= \frac{7}{10} \int \frac{1}{5x^2+16} d(5x^2+16) \\ &= \frac{7}{10} \ln(5x^2+16) + C\end{aligned}$$

d Bepaal $\int 5w^3 \sqrt{w^2-4} dw$.

Stel $t = w^2 - 4$. Uit $t = w^2 - 4$ volgt $w^2 = t + 4$ en $dt = 2w dw$, zodat $w dw = \frac{1}{2} dt$. We schrijven w^3 als $w^2 w$. Na deze voorbereiding is de integraal te bepalen:

$$\begin{aligned}\int 5w^3 \sqrt{w^2-4} dw &= 5 \int w^2 \sqrt{w^2-4} w dw \\ &= \frac{5}{2} \int (t+4) \sqrt{t} dt \\ &= \frac{5}{2} \int \left(t^{\frac{3}{2}} + 4t^{\frac{1}{2}} \right) dt \\ &= t^2 \sqrt{t} + \frac{20}{3} t \sqrt{t} + C \\ &= (w^2-4)^2 \sqrt{w^2-4} + \frac{20}{3} (w^2-4) \sqrt{w^2-4} + C\end{aligned}$$

e Bereken $\int_1^3 \frac{1}{p(1+\sqrt{p})} dp$

Stel $x = \sqrt{p}$, hieruit volgt $p = x^2$ en $dp = 2x dx$.

Grenzen: $p=1 \rightarrow x=1$ en $p=3 \rightarrow x=\sqrt{3}$.

Uitvoeren van de substitutie levert op:

$$\begin{aligned}\int_1^3 \frac{1}{p(1+\sqrt{p})} dp &= 2 \int_1^{\sqrt{3}} \frac{x}{x^2(1+x)} dx \\ &= 2 \int_1^{\sqrt{3}} \frac{1}{x(1+x)} dx\end{aligned}$$

Deze integraal is te berekenen met behulp van breuksplitsing.

Breuksplitsen leidt tot: $\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$. Hieruit volgt:

$$1 = A(1+x) + Bx$$

Kies $x=0$: $1 = A \Rightarrow A=1$

Kies $x=-1$: $1 = -B \Rightarrow B=-1$

Er volgt:

$$\begin{aligned}
2 \int_1^{\sqrt{3}} \frac{1}{x(1+x)} dx &= 2 \int_1^{\sqrt{3}} \frac{1}{x} dx - 2 \int_2^{\sqrt{3}} \frac{1}{x+1} dx \\
&= 2 \int_1^{\sqrt{3}} \frac{1}{x} dx - 2 \int_2^{\sqrt{3}} \frac{1}{x+1} d(x+1) \\
&= 2 [\ln|x|]_1^{\sqrt{3}} - 2 [\ln|x+1|]_1^{\sqrt{3}} \\
&= 2 \ln(\sqrt{3}) - 2 \ln(1+\sqrt{3}) + 2 \ln(2) \\
&= \ln(3) - 2 \ln(1+\sqrt{3}) + 2 \ln(2)
\end{aligned}$$

f Bepaal $\int \sin(5t) \cos(3t) dt$

$$\begin{aligned}
\int \sin(5t) \cos(3t) dt &= \frac{1}{2} \int (\sin(8t) + \sin(2t)) dt \\
&= \frac{1}{2} \int \sin(8t) dt + \frac{1}{2} \int \sin(2t) dt \\
&= \frac{1}{16} \int \sin(8t) d(8t) + \frac{1}{4} \int \sin(2t) d(2t) \\
&= -\frac{1}{16} \cos(8t) - \frac{1}{4} \cos(2t) + C
\end{aligned}$$

g Bepaal $\int 5t^3 \sin^2(2t^4) dt$

Gebruik $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$, zodat $\sin^2(2t^4) = \frac{1}{2}(1 - \cos(4t^4))$

Er volgt:

$$\begin{aligned}
\int 5t^3 \sin^2(2t^4) dt &= \frac{5}{2} \int t^3 (1 - \cos(4t^4)) dt \\
&= \frac{5}{2} \int t^3 dt - \frac{5}{2} \int t^3 \cos(4t^4) dt \\
&= \frac{5}{2} \int t^3 dt - \frac{5}{2} \cdot \frac{1}{16} \int \cos(4t^4) d(4t^4) \\
&= \frac{5}{8} t^4 - \frac{5}{32} \sin(4t^4) + C
\end{aligned}$$

h Bereken $\int_4^9 \frac{1}{(y-1)\sqrt{y}} dy$.

Stel $p = \sqrt{y}$, hieruit volgt $y = p^2$ en $dy = 2pdp$.

Grenzen: $y = 4 \rightarrow p = 2$ en $y = 9 \rightarrow p = 3$.

Uitvoeren van de substitutie levert op:

$$\begin{aligned}
\int_4^9 \frac{1}{(y-1)\sqrt{y}} dy &= 2 \int_2^3 \frac{p}{(p^2-1)p} dp \\
&= 2 \int_2^3 \frac{1}{p^2-1} dp
\end{aligned}$$

Deze integraal is te berekenen met behulp van breuksplitsing.

De noemer is te ontbinden als $p^2 - 1 = (p-1)(p+1)$.

Breuksplitsen leidt tot: $\frac{2}{(p-1)(p+1)} = \frac{A}{p-1} + \frac{B}{p+1}$. Hieruit volgt:

$$2 = A(p+1) + B(p-1)$$

Kies $p = 1$: $2 = 2A \Rightarrow A = 1$

Kies $p = -1$: $2 = -2B \Rightarrow B = -1$

Er volgt:

$$\begin{aligned} 2 \int_2^3 \frac{1}{p^2-1} dp &= \int_2^3 \frac{1}{p-1} dp - \int_2^3 \frac{1}{p+1} dp \\ &= \int_2^3 \frac{1}{p-1} d(p-1) - \int_2^3 \frac{1}{p+1} d(p+1) \\ &= [\ln|p-1|]_2^3 - [\ln|p+1|]_2^3 \\ &= \ln(2) - \ln(4) + \ln(3) \\ &= \ln\left(\frac{3}{2}\right) \end{aligned}$$

Opgave 3

a Bereken $\int_0^{\frac{1}{4}\pi} \frac{p}{\cos^2 p} dp$.

$$\begin{aligned} \int_0^{\frac{1}{4}\pi} \frac{p}{\cos^2 p} dp &= \int_0^{\frac{1}{4}\pi} p d \tan p \\ &= [p \tan p]_0^{\frac{1}{4}\pi} - \int_0^{\frac{1}{4}\pi} \tan p dp \\ &= \frac{1}{4}\pi - \int_0^{\frac{1}{4}\pi} \frac{\sin p}{\cos p} dp \\ &= \frac{1}{4}\pi + \int_0^{\frac{1}{4}\pi} \frac{1}{\cos p} d \cos p \\ &= \frac{1}{4}\pi + [\ln|\cos p|]_0^{\frac{1}{4}\pi} \\ &= \frac{1}{4}\pi + \ln\left|\cos \frac{1}{4}\pi\right| - \ln|\cos 0| \\ &= \frac{1}{4}\pi + \ln \frac{1}{2} \sqrt{2} \\ &= \frac{1}{4}\pi - \frac{1}{2} \ln 2 \end{aligned}$$

b Bepaal $\int \frac{1}{x^3 + 5x^2} dx$.

Oplissing:

De noemer is te ontbinden als $x^3 + 5x^2 = x^2(x+5)$.

Breuksplitsen leidt tot: $\frac{1}{x^2(x+5)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+5}$. Hieruit volgt:

$$1 = Ax(x+2) + B(x+5) + Cx^2$$

Kies $x = 0$: $1 = 5B \Rightarrow B = \frac{1}{5}$

Kies $x = -5$: $1 = 25C \Rightarrow C = \frac{1}{25}$

Coëfficiënt van x^2 : $0 = A + C \Rightarrow A = -\frac{1}{25}$.

Er volgt:

$$\begin{aligned} \int \frac{1}{x^3 + 5x^2} dx &= -\frac{1}{25} \int \frac{1}{x} dx + \frac{1}{5} \int \frac{1}{x^2} dx + \frac{1}{25} \int \frac{1}{x+5} dx \\ &= -\frac{1}{25} \ln|x| - \frac{1}{5x} + \frac{1}{25} \ln|x+5| + C \end{aligned}$$

c Bepaal $\int \frac{5x}{x^2 + 2x + 5} dx$

$$\begin{aligned} \int \frac{5x}{x^2 + 2x + 5} dx &= \frac{5}{2} \int \frac{2x+2}{x^2 + 2x + 5} dx - 5 \int \frac{1}{x^2 + 2x + 5} dx \\ &= \frac{5}{2} \int \frac{1}{x^2 + 2x + 5} d(x^2 + 2x + 5) - 5 \int \frac{1}{(x+1)^2 + 4} d(x+1) \\ &= \frac{5}{2} \ln(x^2 + 2x + 5) - \frac{5}{2} \arctan\left(\frac{x+1}{2}\right) + C \end{aligned}$$

d Bereken $\int_1^2 \frac{x+1}{x^3 + 6x^2 + 9x} dx$

De noemer is te ontbinden als $x^3 + 6x^2 + 9x = x(x+3)^2$.

Breuksplitsen leidt tot: $\frac{x+1}{x(x+3)^3} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$. Hieruit volgt:

$$x+1 = A(x+3)^2 + Bx(x+3) + Cx$$

Kies $x = -3$: $-2 = -3C \Rightarrow C = \frac{2}{3}$

Kies $x = 0$: $1 = 9A \Rightarrow A = \frac{1}{9}$

Coëfficiënt van x^2 : $0 = A + B \Rightarrow B = -\frac{1}{9}$.

Er volgt:

$$\begin{aligned}
\int_1^2 \frac{x+1}{x^3+6x^2+9x} dx &= \frac{1}{9} \int_1^2 \frac{1}{x} dx - \frac{1}{9} \int_2^3 \frac{1}{x+3} dx + \frac{2}{3} \int_2^3 \frac{1}{(x+3)^2} dx \\
&= \frac{1}{9} \int_1^2 \frac{1}{x} dx - \frac{1}{9} \int_2^3 \frac{1}{x+3} d(x+3) + \frac{2}{3} \int_2^3 (x+3)^{-2} d(x+3) \\
&= \frac{1}{9} [\ln|x|]_1^2 - \frac{1}{9} [\ln|x+3|]_2^3 - \frac{2}{3} \left[\frac{1}{x+3} \right]_2^3 \\
&= \frac{1}{9} \ln(2) - \frac{1}{9} \ln(5) + \frac{1}{9} \ln(4) - \frac{2}{3} \left(\frac{1}{5} - \frac{1}{4} \right) \\
&= \frac{1}{3} \ln(2) - \frac{1}{9} \ln(5) + \frac{1}{30}
\end{aligned}$$

e Bepaal $\int \arctan(3x) dx$

We voeren direct partiële integratie uit volgens de formule

$$\int f(x) g'(x) dx = f(x) g(x) - \int g(x) f'(x) dx,$$

waarbij $f(x) = \arctan 3x$ en $g(x) = x$:

$$\begin{aligned}
\int \arctan(3x) dx &= x \arctan(3x) - \int x d(\arctan(3x)) \quad \left(d(\arctan 3x) = \frac{3}{1+9x^2} dx \right) \\
&= x \arctan(3x) - \int \frac{3x}{1+9x^2} dx \\
&= x \arctan(3x) - \frac{1}{6} \int \frac{(1+9x^2)'}{1+9x^2} dx \\
&= x \arctan(3x) - \frac{1}{6} \int \frac{1}{1+9x^2} d(1+9x^2) \\
&= x \arctan(3x) - \frac{1}{6} \ln(1+9x^2) + C
\end{aligned}$$

f Bereken $\int_0^2 z \cdot 3^{4z} dz$.

We moeten 3^{4z} achter het d-teken brengen. Om 3^{4z} achter het d-teken te kunnen brengen, bepalen we eerst $\int 3^{4z} dz$:

$$\int 3^{4z} dz = \frac{1}{4} \int 3^{4z} d(4z) = \frac{1}{4 \ln 3} 3^{4z} + C, \text{ zodat } 3^{4z} dz = d\left(\frac{1}{4 \ln 3} 3^{4z}\right) = \frac{1}{4 \ln 3} d(3^{4z}).$$

Er volgt:

$$\begin{aligned}
\int_0^2 z \cdot 3^{4z} dz &= \frac{1}{4 \ln 3} \int_0^2 z d(3^{4z}) \\
&= \left[\frac{1}{4 \ln 3} z \cdot 3^{4z} \right]_0^2 - \frac{1}{4 \ln 3} \int_0^2 3^{4z} dz \\
&= \frac{2 \cdot 3^8}{4 \ln 3} - \left(\frac{1}{4 \ln 3} \right)^2 \left[3^{4z} \right]_0^2 \\
&= \frac{3^8}{2 \ln 3} - \frac{1}{16 (\ln 3)^2} (3^8 - 1) \\
&= \frac{6561}{2 \ln 3} - \frac{410}{(\ln 3)^2} \\
&= \frac{6561 \cdot \ln 3410 - 820}{2 (\ln 3)^2}
\end{aligned}$$

g Bereken $\int_5^9 \frac{x+8}{x^3 - 6x^2 + 8x} dx$.

Uitwerking:

De noemer is te ontbinden als $x^3 - 6x^2 + 8x = x(x-4)(x-2)$.

Breuksplitsen leidt tot: $\frac{x+8}{x(x-4)(x-2)} = \frac{A}{x} + \frac{B}{x-4} + \frac{C}{x-2}$. Hieruit volgt:

$$x+8 = A(x-4)(x-2) + Bx(x-2) + Cx(x-4)$$

Kies $x=4$: $12 = 8B \Rightarrow B = \frac{3}{2}$

Kies $x=2$: $10 = -4C \Rightarrow C = -\frac{10}{4} = -\frac{5}{2}$

Kies $x=0$: $8 = 8A \Rightarrow A=1$

Er volgt:

$$\begin{aligned}
\int_5^9 \frac{x+8}{x^3 - 6x^2 + 8x} dx &= \int_5^9 \frac{1}{x} dx + \frac{3}{2} \int_5^9 \frac{1}{x-4} dx - \frac{5}{2} \int_5^9 \frac{1}{x-2} dx \\
&= \int_5^9 \frac{1}{x} dx + \frac{3}{2} \int_5^9 \frac{1}{x-4} d(x-4) - \frac{5}{2} \int_5^9 \frac{1}{x-2} d(x-2) \\
&= \left[\ln|x| \right]_5^9 + \frac{3}{2} \left[\ln|x-4| \right]_5^9 - \frac{5}{2} \left[\ln|x-2| \right]_5^9 \\
&= \ln(9) - \ln(5) + \frac{3}{2} \ln(5) - \frac{5}{2} \ln(7) + \frac{5}{2} \ln(3) \\
&= \frac{9}{2} \ln(3) + \frac{1}{2} \ln(5) - \frac{5}{2} \ln(7)
\end{aligned}$$

h Bepaal $\int 8t^2 \sin(4t) dt$.

We moeten de factor $\sin(4t)$ achter het d-teken brengen. We bepalen eerst

apart $\int \sin(4t) dt$:

$$\int \sin(4t) dt = \frac{1}{4} \int \sin(4t) d(4t) = -\frac{1}{4} \cos(4t) + C, \text{ zodat } \sin(4t) dt = -\frac{1}{4} d(\cos(4t)).$$

Er volgt:

$$\begin{aligned} \int 8t^2 \sin(4t) dt &= -2 \int t^2 d(\cos(4t)) \\ &= -2t^2 \cos(4t) + 2 \int \cos(4t) d(t^2) \\ &= -2t^2 \cos(4t) + 4 \int t \cos(4t) dt \\ &= -2t^2 \cos(4t) + \int t d(\sin(4t)) \\ &= -2t^2 \cos(4t) + t \sin(4t) - \int \sin(4t) dt \\ &= -2t^2 \cos(4t) + t \sin(4t) + \frac{1}{4} \cos(4t) + C \end{aligned}$$

Opgave 4

a Noem de integrand $f(x) = e^{\sin x}$

Deelintervallengte $h = \frac{3-1}{4} = \frac{1}{2}$. Tabel van functiewaarden:

x	$x_0 = 1$	$x_1 = 1,5$	$x_2 = 2$	$x_3 = 2,5$	$x_4 = 3$
$f(x)$	$e^{\sin 1}$	$e^{\sin 1,5}$	$e^{\sin 2}$	$e^{\sin 2,5}$	$e^{\sin 3}$

We vinden:

$$\begin{aligned} T_4 &= \frac{1}{2} h \{ f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4) \} \\ &= \frac{1}{4} (e^{\sin 1} + 2 \cdot e^{\sin 1,5} + 2 \cdot e^{\sin 2} + 2 \cdot e^{\sin 2,5} + e^{\sin 3}) \\ &= 4.374532783 \end{aligned}$$

b

$$\begin{aligned} S_4 &= \frac{1}{3} h \{ f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4) \} \\ &= \frac{1}{6} (e^{\sin 1} + 4 \cdot e^{\sin 1,5} + 2 \cdot e^{\sin 2} + 4 \cdot e^{\sin 2,5} + e^{\sin 3}) \\ &= 4.426627858 \end{aligned}$$

c Als we $I = \int_1^3 e^{\sin x} dx$ benaderen met de trapeziumregel toegepast op 4 deelintervallen,

dan geldt voor de maximale fout in absolute waarde $|E_T| \leq \frac{(3-1)^3}{12 \cdot 4^2} \max_{1 \leq x \leq 3} |f''(x)|$

De tweede afgeleide van $f(x) = e^{\sin x}$: $f''(x) = -\sin(x)e^{\sin x} + \cos^2(x)e^{\sin x}$.

Voor het berekenen van het gevraagde maximum berekenen we de derde afgeleide:

$$\begin{aligned} f'''(x) &= -\cos(x)e^{\sin x} - 3\sin(x)\cos(x)e^{\sin x} + \cos^3(x)e^{\sin x} \\ &= \cos(x)e^{\sin x} (-1 - 3\sin(x) + \cos^2(x)) \end{aligned}$$

$$f'''(x) = 0 \Leftrightarrow \cos(x) = 0 \vee \cos^2(x) - 1 - 3\sin(x) = 0$$

Opgelost wordt: $\cos(x) = 0$ en $\cos^2(x) - 1 - 3\sin(x) = 0$:

$$\cos(x) = 0 \Rightarrow x = \frac{1}{2} \pi + k\pi, \text{ met } k \in \mathbb{Z}$$

$$\begin{aligned}\cos^2(x) - 1 - 3\sin(x) = 0 &\Rightarrow 1 - \sin^2(x) - 1 - 3\sin(x) = 0 \\ &\Rightarrow \sin^2(x) + 3\sin(x) = 0 \Rightarrow \sin(x)(\sin(x) + 3) = 0 \\ &\Rightarrow \sin(x) = 0 \Rightarrow x = k \cdot \pi, \text{ met } k \in \mathbb{Z}\end{aligned}$$

Het maximum van $|f''(x)|$ op het interval $[1, 3]$ wordt bereikt voor $x = \frac{1}{2}\pi$ of op de rand van het interval.

Vergelijken van de waarden geeft:

$$f''(1) = -1,2748, \quad f''(3) = 0,9661 \quad \text{en} \quad f''\left(\frac{1}{2}\pi\right) = -e,$$

zodat $\max_{1 \leq x \leq 3} |f''(x)| = e$.

$$\text{Er volgt: } |E_T| \leq \frac{(3-1)^3 \cdot e}{12 \cdot 4^2} = \frac{1}{24}e$$

d Er moet gelden

$$|E_T| \leq \frac{2^3}{12 \cdot n^2} \max_{1 \leq x \leq 3} |f''(x)| < \frac{1}{2} \cdot 10^{-6}$$

Er volgt, met $\max_{1 \leq x \leq 3} |f''(x)| = e$:

$$\frac{2^3}{12 \cdot n^2} \cdot e < \frac{1}{2} \cdot 10^{-6} \Rightarrow n > \sqrt{\frac{2^4 \cdot 10^6 \cdot e}{12}} = 1903,8$$

Het aantal deelintervallen moeten we minimaal $n = 1904$ nemen om er zeker van te zijn dat I in 6 decimalen nauwkeurig benaderd wordt.

Opgave 5

a

$$\begin{aligned}\int_0^3 \frac{e^x}{x^2} dx &= \lim_{p \downarrow 0} \int_p^3 \frac{e^x}{x^2} dx \\ &= \lim_{p \downarrow 0} \left[\int_p^3 \frac{e^x}{x^2} d\left(\frac{1}{x}\right) \right] \\ &= \lim_{p \downarrow 0} \left[\frac{1}{x} e^x \right]_p^3 = \lim_{p \downarrow 0} \left(-e^{\frac{1}{3}} + e^{\frac{1}{p}} \right) = \infty\end{aligned}$$

De integraal divergeert.

b

$$\begin{aligned}\int_0^3 \frac{e^{-\frac{1}{x}}}{x^2} dx &= \lim_{p \downarrow 0} \int_p^3 \frac{e^{-\frac{1}{x}}}{x^2} dx \\ &= \lim_{p \downarrow 0} \int_p^3 \frac{e^{-\frac{1}{x}}}{x^2} d\left(-\frac{1}{x}\right) \\ &= \lim_{p \downarrow 0} \left[e^{-\frac{1}{x}} \right]_p^3 \\ &= \lim_{p \downarrow 0} \left(e^{-\frac{1}{3}} - e^{-\frac{1}{p}} \right) = e^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{e}}\end{aligned}$$

De integraal convergeert naar $e^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{e}}$.

c
$$\int_{-\infty}^{-2} \frac{4}{x-1} dx = \lim_{b \rightarrow -\infty} \int_b^{-2} \frac{4}{x-1} dx = \lim_{b \rightarrow -\infty} [4 \ln|x-1|]_b^{-2} = \lim_{b \rightarrow -\infty} (4 \ln 3 - 4 \ln|b-1|) = -\infty$$

d

$$\begin{aligned}\int_{-\infty}^{\infty} \frac{e^x}{e^{2x} + 1} dx &= \lim_{s \rightarrow -\infty} \int_s^0 \frac{1}{(e^x)^2 + 1} d(e^x) + \lim_{t \rightarrow \infty} \int_0^t \frac{1}{(e^x)^2 + 1} d(e^x) \\ &= \lim_{s \rightarrow -\infty} [\arctan(e^x)]_s^0 + \lim_{t \rightarrow \infty} [\arctan(e^x)]_0^t \\ &= \lim_{s \rightarrow -\infty} (\arctan 1 - \arctan(e^s)) + \lim_{t \rightarrow \infty} (\arctan(e^t) - \arctan 1) \\ &= \frac{1}{4}\pi - 0 + \frac{1}{2}\pi - \frac{1}{4}\pi = \frac{1}{2}\pi\end{aligned}$$

De integraal convergeert naar $\frac{1}{2}\pi$.

Opgave 6

De oppervlakte van het meer is:

$$O = \int_0^{240} b(x) dx, \text{ met } b = b(x) \text{ de breedte van het meer op de plaats } x.$$

a

$$\begin{aligned}T_{12} &= \frac{1}{2} \cdot 20 \{b(0) + 2 \cdot b(20) + 2 \cdot b(40) + 2 \cdot b(60) + \dots + 2 \cdot b(200) + 2 \cdot b(220) + b(240)\} \\ &= 10 \{0 + 2 \cdot 80 + 2 \cdot 84 + 2 \cdot 72 + 2 \cdot 51 + 2 \cdot 42 + 2 \cdot 47 + 2 \cdot 59 + 2 \cdot 81 + 2 \cdot 115 + 2 \cdot 124 + 2 \cdot 86 + 0\} \\ &= 16820 \text{ m}^2\end{aligned}$$

b

$$\begin{aligned}S_{12} &= \frac{1}{3} \cdot 20 \{b(0) + 4 \cdot b(20) + 2 \cdot b(40) + 4 \cdot b(60) + \dots + 2 \cdot b(200) + 4 \cdot b(220) + b(240)\} \\ &= \frac{1}{3} \cdot 20 \{0 + 4 \cdot 80 + 2 \cdot 84 + 4 \cdot 72 + 2 \cdot 51 + 4 \cdot 42 + 2 \cdot 47 + 4 \cdot 59 + 2 \cdot 81 + 4 \cdot 115 + 2 \cdot 124 + 4 \cdot 86 + 0\} \\ &= 17266,667 \text{ m}^2\end{aligned}$$