Uitwerkingen extra opgaven hoofdstuk 4 Limieten en differentiaalrekening

1.

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{2x+1}{3x-1} = \frac{2}{3} \text{(via een tabel te bepalen)}$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} \frac{2x+1}{3x-1} = \frac{2}{3} \text{(via een tabel te bepalen)}$$

Dit betekent dat de grafiek van f zowel naar links als naar rechts een horizontale asymptoot heeft. De vergelijking van deze asymptoot is in beide gevallen $x = \frac{2}{3}$.

2.

a. $\lim_{x \to -3} \dfrac{x-3}{x^2+9} = \dfrac{-6}{9} = -\dfrac{2}{3}$

b. $\lim_{x \to -3} \dfrac{x-3}{(2x+6)^2} = \lim_{x \to -3} \left(\dfrac{-6}{0^2} \right) = -\infty$

c. $\lim_{x \to -3} \dfrac{x-3}{x^2-9} = \lim_{x \to -3} \dfrac{x-3}{(x-3)(x+3)} = \lim_{x \to -3} \dfrac{1}{x+3}$

   Deze limiet bestaat niet, want $\lim_{x \uparrow -3} \dfrac{1}{x+3} = -\infty$ en $\lim_{x \downarrow -3} \dfrac{1}{x+3} = +\infty$

d. $\lim_{x \to -3} \dfrac{x+3}{x^2-9} = \lim_{x \to -3} \dfrac{x+3}{(x+3)(x-3)} = \lim_{x \to -3} \dfrac{1}{x-3} = \dfrac{1}{-3-3} = -\dfrac{1}{6}$

e. $\lim_{x \uparrow -3} \dfrac{x^2+9}{x^2-9} = \left(\dfrac{18}{0^+} \right) = +\infty$

f. $\lim_{x \downarrow 3} \dfrac{x^2+9}{x^2-9} = \left(\dfrac{18}{0^+} \right) = +\infty$

3.

a. $\lim_{x \downarrow 0} \dfrac{x^2+4x}{x^2-4x} = \lim_{x \downarrow 0} \dfrac{x(x+4)}{x(x-4)} = \lim_{x \downarrow 0} \dfrac{x+4}{x-4} = \dfrac{4}{-4} = -1$

b. $\lim_{x \uparrow 0} \dfrac{x^2+4x}{x^2-4x} = \lim_{x \uparrow 0} \dfrac{x(x+4)}{x(x-4)} = \lim_{x \uparrow 0} \dfrac{x+4}{x-4} = \dfrac{4}{-4} = -1$

c. $\lim_{x \to 0} \dfrac{x^2+4x}{x^2-4x} = \lim_{x \to 0} \dfrac{x(x+4)}{x(x-4)} = \lim_{x \to 0} \dfrac{x+4}{x-4} = \dfrac{4}{-4} = -1$

d. $\lim_{x \downarrow 4} \dfrac{x^2+4x}{x^2-4x} = \lim_{x \downarrow 4} \dfrac{x(x+4)}{x(x-4)} = \lim_{x \downarrow 4} \dfrac{x+4}{x-4} = \dfrac{8}{0^+} = +\infty$

e. $\lim_{x \uparrow 4} \dfrac{x^2+4x}{x^2-4x} = \lim_{x \uparrow 4} \dfrac{x(x+4)}{x(x-4)} = \lim_{x \uparrow 4} \dfrac{x+4}{x-4} = \dfrac{8}{0^-} = -\infty$

f. $\lim_{x \to 4} \dfrac{x^2+4x}{x^2-4x}$ bestaat niet, want $\lim_{x \downarrow 4} \dfrac{x^2+4x}{x^2-4x} = +\infty$ en $\lim_{x \uparrow 4} \dfrac{x^2+4x}{x^2-4x} = -\infty$

g. $\displaystyle\lim_{x\uparrow 2}\frac{x^2+2x}{x^2-4}=\lim_{x\uparrow 2}\frac{x(x+2)}{(x-2)(x+2)}=\lim_{x\uparrow 2}\frac{x}{x-2}=\frac{2}{0^-}=-\infty$

h. $\displaystyle\lim_{x\downarrow 2}\frac{x^2-2x}{x^2-4}=\lim_{x\downarrow 22}\frac{x(x+2)}{(x-2)(x+2)}=\lim_{x\downarrow 2}\frac{x}{x-2}=\frac{2}{0^+}=+\infty$

i. $\displaystyle\lim_{x\uparrow 2}\frac{x^2+2x}{(x-2)^2}=\lim_{x\uparrow 2}\frac{4+4}{0^+}=+\infty$

j. $\displaystyle\lim_{x\downarrow 2}\frac{x^2+2x}{(x-2)^2}=\lim_{x\downarrow 2}\frac{4+4}{0^+}=+\infty$

k. $\displaystyle\lim_{x\uparrow 2}\frac{x^2-2x}{(x-2)^2}=\lim_{x\uparrow 2}\frac{x(x-2)}{(x-2)^2}=\lim_{x\uparrow 2}\frac{x}{x-2}=\frac{2}{0^-}=-\infty$

l. $\displaystyle\lim_{x\downarrow 2}\frac{x^2-2x}{(x-2)^2}=\lim_{x\downarrow 2}\frac{x(x-2)}{(x-2)^2}=\lim_{x\downarrow 2}\frac{x}{x-2}=\frac{2}{0^+}=+\infty$

4.

a. $\displaystyle\lim_{t\to\infty}\frac{4t^3-1}{3t^2+2}=\lim_{t\to\infty}\frac{4\frac{t^3}{t^2}-\frac{1}{t^2}}{3\frac{t^2}{t^2}+\frac{2}{t^2}}=\lim_{t\to\infty}\frac{4t-\frac{1}{t^2}}{3+\frac{2}{t^2}}=\frac{\infty-0}{3+0}=\infty$

b. $\displaystyle\lim_{t\to-\infty}\frac{4t^3+1}{3t^2-2}=\lim_{t\to-\infty}\frac{4\frac{t^3}{t^2}+\frac{1}{t^2}}{3\frac{t^2}{t^2}-\frac{2}{t^2}}=\lim_{t\to-\infty}\frac{4t+\frac{1}{t^2}}{3-\frac{2}{t^2}}=\frac{-\infty+0}{3-0}=-\infty$

c. $\displaystyle\lim_{t\to\infty}\frac{4t^2-1}{3t^2+2}=\lim_{t\to\infty}\frac{4\frac{t^2}{t^2}-\frac{1}{t^2}}{3\frac{t^2}{t^2}+\frac{2}{t^2}}=\lim_{t\to\infty}\frac{4-\frac{1}{t^2}}{3+\frac{2}{t^2}}=\frac{4-0}{3+0}=\frac{4}{3}$

d. $\displaystyle\lim_{t\to-\infty}\frac{4t^2+1}{3t^2-2}=\lim_{t\to-\infty}\frac{4\frac{t^2}{t^2}+\frac{1}{t^2}}{3\frac{t^2}{t^2}-\frac{2}{t^2}}=\lim_{t\to-\infty}\frac{4+\frac{1}{t^2}}{3-\frac{2}{t^2}}=\frac{4+0}{3-0}=\frac{4}{3}$

e. $\displaystyle\lim_{t\to\infty}\frac{4t^3-1}{3t^4+2}=\lim_{t\to\infty}\frac{4\frac{t^3}{t^4}-\frac{1}{t^4}}{3\frac{t^4}{t^4}+\frac{2}{t^4}}=\lim_{t\to\infty}\frac{\frac{4}{t}-\frac{1}{t^4}}{3+\frac{2}{t^4}}=\frac{0-0}{3+0}=\frac{0}{3}=0$

f. $\displaystyle\lim_{t\to-\infty}\frac{4t^3-1}{3t^4+2}=\lim_{t\to-\infty}\frac{4\frac{t^3}{t^4}-\frac{1}{t^4}}{3\frac{t^4}{t^4}+\frac{2}{t^4}}=\lim_{t\to-\infty}\frac{\frac{4}{t}-\frac{1}{t^4}}{3+\frac{2}{t^4}}=\frac{0-0}{3+0}=\frac{0}{3}=0$

g.

$$\lim_{u\to\infty}\frac{1+2\sqrt{u}-4u^4}{2u^2+\sqrt{u}+5}=\lim_{u\to\infty}\frac{\dfrac{1}{u^2}+2\dfrac{\sqrt{u}}{u^2}-4\dfrac{u^4}{u^2}}{2\dfrac{u^2}{u^2}+\dfrac{\sqrt{u}}{u^2}+\dfrac{5}{u^2}}=\lim_{u\to\infty}\frac{\dfrac{1}{u^2}+2\dfrac{1}{u\sqrt{u}}-4u^2}{2+\dfrac{1}{u\sqrt{u}}+\dfrac{5}{u^2}}$$

$$=\lim_{u\to\infty}\frac{0+2\cdot0-4\cdot\infty}{2+0+5\cdot0}=\frac{-\infty}{2}=-\infty$$

h.

$$\lim_{u\to\infty}\frac{3+4u^2}{5-2u^2+u\sqrt{u}}=\lim_{u\to\infty}\frac{\dfrac{3}{u^2}+4\dfrac{u^2}{u^2}}{\dfrac{5}{u^2}-2\dfrac{u^2}{u^2}+\dfrac{u\sqrt{u}}{u^2}}$$

$$=\lim_{u\to\infty}\frac{\dfrac{3}{u^2}+4}{\dfrac{5}{u^2}-2+\dfrac{1}{\sqrt{u}}}=\frac{0+4}{0-2+0}=\frac{4}{-2}=-2$$

5.

a. $\quad\lim_{x\to0}\dfrac{\sin(-3x)}{5x}=\lim_{x\to0}\dfrac{-\sin(3x)}{5x}\lim_{x\to0}-\dfrac{3}{5}\cdot\dfrac{\sin(3x)}{3x}=-\dfrac{3}{5}\cdot1=-\dfrac{3}{5}$

b. $\quad\lim_{x\uparrow0}\dfrac{\tan\left(\frac{1}{2}x\right)}{2x}=\lim_{x\uparrow0}\dfrac{\frac{1}{2}}{2}\cdot\dfrac{\tan\left(\frac{1}{2}x\right)}{\frac{1}{2}x}=\dfrac{1}{4}\cdot1=\dfrac{1}{4}$

c. $\quad\lim_{x\uparrow0}\dfrac{x}{\cos(2x)}=\dfrac{0}{1}=0$

d.

$$\lim_{x\to0}\frac{x\sin(2x)}{\tan^2(3x)}=\lim_{x\to0}\left(\frac{3x}{\tan(3x)}\cdot\frac{1}{3}\right)\left(\frac{\sin(2x)}{2x}\cdot\frac{2x}{1}\right)\left(\frac{3x}{\tan(3x)}\cdot\frac{1}{3x}\right)$$

$$=\lim_{x\to0}\frac{3x}{\tan(3x)}\frac{\sin(2x)}{2x}\frac{3x}{\tan(3x)}\cdot\frac{1}{3}\cdot\frac{2x}{1}\cdot\frac{1}{3x}$$

$$=\lim_{x\to0}\frac{3x}{\tan(3x)}\frac{\sin(2x)}{2x}\frac{3x}{\tan(3x)}\cdot\frac{2}{9}=1\cdot1\cdot1\cdot\frac{2}{9}=\frac{2}{9}$$

e. $\quad\lim_{x\to0}\dfrac{3x^2}{\sin x\cos x}=\lim_{x\to0}\dfrac{x}{\sin x}\dfrac{3x}{\cos x}=1\cdot\dfrac{0}{1}=0$

f.

$$\lim_{x\downarrow0}\frac{5x^2}{(\sin(3x))^2\tan(2x)}=\lim_{x\downarrow0}5\cdot\left(\frac{3x}{\sin(3x)}\cdot\frac{1}{3}\right)\left(\frac{3x}{\sin(3x)}\cdot\frac{1}{3}\right)\left(\frac{2x}{\tan(2x)}\cdot\frac{1}{2x}\right)$$

$$=\lim_{x\downarrow0}\left(\frac{3x}{\sin(3x)}\right)\left(\frac{3x}{\sin(3x)}\right)\left(\frac{2x}{\tan(2x)}\right)\cdot5\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{2x}$$

$$=1\cdot1\cdot1\cdot5\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\infty=\infty$$

6.

a. $\lim\limits_{x\to\infty} \dfrac{\tan(3x)}{2x}$ bestaat niet, want $\tan(3x)$ gaat voor groter wordende waarden van x afwisselend naar $+\infty$ en $-\infty$.

b. Omdat $-1 \le \sin(3x) \le 1$ geldt $\dfrac{-1}{x^2} \le \dfrac{\sin(3x)}{x^2} \le \dfrac{1}{x^2}$. Ook geldt: $\lim\limits_{x\to\infty} \dfrac{1}{x^2} = 0$.

Vanwege de insluitstelling geldt dan: $\lim\limits_{x\to\infty} \dfrac{\sin(3x)}{x^2} = 0$

c. $\lim\limits_{x\to\infty} \dfrac{x}{\sin x}$ bestaat niet. Voor groter wordende waarden van x neemt

$\sin x$ afwisselend positieve en negatieve waarden aan, waardoor $\dfrac{x}{\sin x}$ steeds

sterker positieve en negatieve waarden aanneemt. Er treden ook steeds verticale asymptoten op.

d. $\lim\limits_{x\to-\infty} \dfrac{x^2}{(\sin x)^3}$ bestaat niet. Voor steeds sterker negatieve waarden van x neemt

$(\sin x)^3$ afwisselend positieve en negatieve waarden aan, waardoor $\dfrac{x^2}{(\sin x)^3}$

steeds sterker positieve en negatieve waarden aanneemt. Er treden ook steeds verticale asymptoten op.

e. $\lim\limits_{x\to-\infty} \dfrac{2^x}{\sin x}$ bestaat niet. Voor groter wordende waarden van x neemt

$\sin x$ afwisselend positieve en negatieve waarden aan en 2^x wordt steeds sterker

positief, waardoor $\dfrac{2^x}{\sin x}$ steeds sterker positieve en negatieve waarden aanneemt.

Er treden ook steeds verticale asymptoten op.

f. Omdat $-1 \le \sin(3x) \le 1$ geldt $\dfrac{-1}{e^{2x}} \le \dfrac{\sin(3x)}{e^{2x}} \le \dfrac{1}{e^{2x}}$.

Ook geldt: $\lim\limits_{x\to\infty} \dfrac{1}{e^{2x}} = \lim\limits_{x\to\infty} e^{-2x} = 0$.

Vanwege de insluitstelling geldt dan: $\lim\limits_{x\to\infty} \dfrac{\sin(3x)}{e^{2x}} = 0$

7.

a.. V.A.: $t = -2$; H.A.: $y = 0$

b. V.A.: geen; H.A.: $y = 3$

c. V.A.: $x = 0$ $x = -\frac{1}{2}$; H.A.: $y = 0$

d. V.A.: $x = 0$, $x = -\frac{1}{2}$ en $x = \frac{1}{2}$; H.A.: $y = 0$

e. V.A.: $u = 0$; H.A.: $y = 2$

f. V.A.: $u = 0$; H.A.: $y = 0$

8.

a. $f(x) = \dfrac{x^2 + 5x + 6}{x + 5}$; oneindige discontinuïteit voor $x = -5$.

b. $f(x) = \dfrac{x^2 + 5x + 6}{x + 3}$; ophefbare discontinuïteit voor $x = -3$;.

c. $f(x) = \dfrac{x + 2}{x^2 + x - 2}$; ophefbare discontinuïteit voor $x = -2$ en

oneindige discontinuïteit voor $x = 1$.

d. $f(x) = \dfrac{x^2 - 1}{|x - 1|}$; eindige sprong discontinuïteit voor $x = 1$.

9.

a. $f(x) = \begin{cases} \dfrac{x^2 - 4}{2x - 4} & \text{voor} \quad x \neq 2 \\ 2 & \text{voor} \quad x = 2 \end{cases}$

Deze functie is nergens discontinu

b.

$f(x) = \begin{cases} x^2 - 2 & \text{voor} \quad x \leq -1 \\ \dfrac{1}{\tan\left(\frac{1}{4}\pi x\right)} & \text{voor} \quad -1 < x < 1 \\ x^2 - 2 & \text{voor} \quad x \geq 1 \end{cases}$

Eindige sprongdiscontinuïteit voor $x = 1$
(N.B. Voor $x = -1$ is de functie wel continu)

10.

a.

$f(x) = \begin{cases} \dfrac{\sin(3x)}{5x} & \text{voor} \quad x \neq 0 \\ a & \text{voor} \quad x = 0 \end{cases}$

Omdat $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \dfrac{\sin(3x)}{5x} = \dfrac{3}{5}$ moet voor a de waarde $\dfrac{3}{5}$ gekozen worden.

b.

$f(x) = \begin{cases} \dfrac{\cos(3x)}{5x} & \text{voor} \quad x \neq 0 \\ a & \text{voor} \quad x = 0 \end{cases}$

$\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \dfrac{\cos(3x)}{5x}$ bestaat niet, omdat $\lim\limits_{x \uparrow 0} f(x) = \lim\limits_{x \to 0} \dfrac{\cos(3x)}{5x} = \dfrac{1}{0^-} = -\infty$ en

$\lim\limits_{x \downarrow 0} f(x) = \lim\limits_{x \downarrow 0} \dfrac{\cos(3x)}{5x} = \dfrac{1}{0^+} = +\infty$. Er is daarom geen enkele waarde van a

waarvoor de functie f continu is.

c.

$$f(x) = \begin{cases} \dfrac{x^3 - x}{|x|} & \text{voor} \quad x \neq 0 \\ a & \text{voor} \quad x = 0 \end{cases}$$

$\lim\limits_{x \to 0} f(x)$ bestaat niet, omdat linker- en rechterlimiet ongelijk zijn aan elkaar:

$$\lim_{x \uparrow 0} f(x) = \lim_{x \uparrow 0} \frac{x^3 - x}{|x|} = \lim_{x \uparrow 0} \frac{x(x^2 - 1)}{-x} = \lim_{x \uparrow 0} \frac{(x^2 - 1)}{-1} = 1 \text{ en}$$

$$\lim_{x \downarrow 0} f(x) = \lim_{x \downarrow 0} \frac{x^3 - x}{|x|} = \lim_{x \downarrow 0} \frac{x(x^2 - 1)}{x} = \lim_{x \downarrow 0} (x^2 - 1) = -1.$$

Er is daarom geen enkele waarde van a waarvoor de functie f continu is.

11.

a. $\quad \lim\limits_{x \to \infty} \arccos(1 - e^{-2x}) = \arccos(1 - 0) = 0$

b. $\quad \lim\limits_{x \downarrow 0} \dfrac{e^{\tan 3x}}{5x} = \left(\dfrac{1}{0^+} \right) = \infty$

c. $\quad \lim\limits_{x \downarrow 0} \dfrac{\tan 3x}{5x} = \lim\limits_{x \downarrow 0} \dfrac{\tan 3x}{3x} \cdot \dfrac{3}{5} = 1 \cdot \dfrac{3}{5} = \dfrac{3}{5}$ en dus $\lim\limits_{x \downarrow 0} e^{\frac{\tan 3x}{5x}} = e^{\frac{3}{5}} \approx 1{,}822$

12.

a. $\quad \lim\limits_{t \downarrow 0} \dfrac{\tan 3t}{2t} = \lim\limits_{t \downarrow 0} \dfrac{\tan 3t}{3t} \cdot \dfrac{3}{2} = \dfrac{3}{2}$ en dus $\lim\limits_{t \downarrow 0} \ln\left(\dfrac{\tan 3t}{2t} \right) = \ln\left(\dfrac{3}{2} \right) \approx 0{,}405$

b. $\quad$ Omdat $-1 \leq \cos(3t) \leq 1$ en $t \geq 0$ (we nemen een limiet voor $t \to \infty$) geldt:

$\dfrac{-1}{2t} \leq \dfrac{\cos 3t}{2t} \leq \dfrac{1}{2t}$. Ook geldt: $\lim\limits_{t \to \infty} \dfrac{1}{2t} = 0$.

Vanwege de insluitstelling geldt dan: $\lim\limits_{t \to \infty} \dfrac{\cos(3t)}{2t} = 0$. Ook geldt: $\left| \dfrac{\cos 3t}{2t} \right| \geq 0$

Daarom is $\lim\limits_{t \to \infty} \ln\left| \dfrac{\cos 3t}{2t} \right| = \ln\left(\lim\limits_{t \to \infty} \left| \dfrac{\cos 3t}{2t} \right| \right) = \ln\left(0^+ \right) = -\infty$

13.

a. $\quad \dfrac{\Delta y}{\Delta x} = \dfrac{f(1) - f(0)}{1 - 0} = \dfrac{0 - 0}{1 - 0} = 0.$

b. $\quad \dfrac{\Delta y}{\Delta x} = \dfrac{f(0{,}1) - f(0)}{0{,}1 - 0} = \dfrac{(0{,}01 - 0{,}1) - 0}{0{,}1 - 0} = \dfrac{-0{,}09}{0{,}1} = -0{,}9$

c. $\quad \dfrac{\Delta y}{\Delta x} = \dfrac{f(3{,}9) - f(4)}{3{,}9 - 4} = \dfrac{13{,}24 - 14}{3{,}9 - 4} = \dfrac{-0{,}76}{-0{,}1} = 7{,}6.$

d. $\quad \dfrac{\Delta y}{\Delta x} = \dfrac{f(0{,}9) - f(1)}{0{,}9 - 1} = \dfrac{-0{,}139 - 0}{0{,}9 - 1} = \dfrac{-0{,}139}{-0{,}1} = 1{,}39.$

14.

a. $\dfrac{\Delta y}{\Delta x} = \dfrac{f(1) - f(0)}{1 - 0} = \dfrac{1 - 3}{1 - 0} = -2$.

b. $\dfrac{\Delta y}{\Delta x} = \dfrac{f(-2) - f(-3)}{(-2) - (-3)} = \dfrac{7 - 9}{(-2) - (-3)} = -2$

c. $\dfrac{\Delta y}{\Delta x} = \dfrac{f(a - 1) - f(a)}{(a - 1) - a} = \dfrac{3 - 2(a - 1) - (3 - 2a)}{(a - 1) - a} = \dfrac{3 - 2a + 2 - 3 + 2a}{-1} = -2$

d.

$$\dfrac{\Delta y}{\Delta x} = \dfrac{f(a + \Delta a) - f(a)}{(a + \Delta a) - a} = \dfrac{3 - 2(a + \Delta a) - (3 - 2a)}{(a + \Delta a) - a} = $$

$$= \dfrac{3 - 2a - 2\Delta a - 3 + 2a}{\Delta a} = \dfrac{-2\Delta a}{\Delta a} = -2$$

15.

a.

$$\dfrac{\Delta y}{\Delta t} = \dfrac{f(t + \Delta t) - f(t)}{\Delta t} = \dfrac{\left(2(t + \Delta t)^2 - 3(t + \Delta t)\right) - \left(2t^2 - 3t\right)}{\Delta t} =$$

$$= \dfrac{\left(2t^2 + 4t\Delta t + 2(\Delta t)^2 - 3t - 3\Delta t\right) - \left(2t^2 - 3t\right)}{\Delta t} =$$

$$= \dfrac{4t\Delta t + 2(\Delta t)^2 - 3\Delta t}{\Delta t} = 4t + 2\Delta t - 3 = 4t - 3 + 2\Delta t$$

b.

$$\dfrac{\Delta z}{\Delta u} = \dfrac{f(u + \Delta u) - f(u)}{\Delta u} = \dfrac{\left((u + \Delta u) + 1\right)^2 - (u + 1)^2}{\Delta u} =$$

$$= \dfrac{\left(u^2 + (\Delta u)^2 + 1^2 + 2u\Delta u + 2u \cdot 1 + 2\Delta u \cdot 1\right) - \left(u^2 + 2u + 1\right)}{\Delta u} =$$

$$= \dfrac{(\Delta u)^2 + 2u\Delta u + 2\Delta u}{\Delta u} = \Delta u + 2u + 2 = 2u + 2 + \Delta u$$

16.

a

$$f'(1) = \lim_{\Delta t \to 0} \dfrac{f(1 + \Delta t) - f(1)}{\Delta t} = \lim_{\Delta t \to 0} \dfrac{\left((1 + \Delta t)^2 - 3(1 + \Delta t)\right) - \left(1^2 - 3 \cdot 1\right)}{\Delta t} =$$

$$= \lim_{\Delta t \to 0} \dfrac{\left(1 + 2\Delta t + (\Delta t)^2 - 3 - 3\Delta t\right) - (-2)}{\Delta t} = \lim_{\Delta t \to 0} \dfrac{-\Delta t + (\Delta t)^2}{\Delta t} =$$

$$= \lim_{\Delta t \to 0} (-1 + \Delta t) = -1$$

b

$$f'(t) = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\left((t + \Delta t)^2 - 3(t + \Delta t)\right) - \left(t^2 - 3t\right)}{\Delta t} =$$

$$= \lim_{\Delta t \to 0} \frac{\left(t^2 + 2t\Delta t + (\Delta t)^2 - 3t - 3\Delta t\right) - \left(t^2 - 3t\right)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{2t\Delta t - 3\Delta t + (\Delta t)^2}{\Delta t} = \lim_{\Delta t \to 0} (2t - 3 + \Delta t) = 2t - 3$$

17.

a. $\quad f(x) = -2x^4 + 3x^2 - 4 \Rightarrow f'(x) = -2 \cdot 4x^3 + 3 \cdot 2x^1 - 0 = -8x^3 + 6x$

b. $\quad f(x) = x\sqrt[4]{x^3} - \frac{8}{3}\sqrt{x} + 3x - 1 = x^{\frac{7}{4}} - \frac{8}{3}x^{\frac{1}{2}} + 3x - 1 \Rightarrow$

$$f'(x) = \frac{7}{4}x^{\frac{3}{4}} - \frac{8}{3} \cdot \frac{1}{2}x^{-\frac{1}{2}} + 3 - 0 = \frac{7}{4}\sqrt[4]{x^3} - \frac{4}{3\sqrt{x}} + 3$$

c. $\quad f(x) = \frac{4}{x^2\sqrt{x}} - \frac{1}{\sqrt{x}} + x^3\sqrt{x} + \sqrt{5} = 4x^{-\frac{5}{2}} - x^{-\frac{1}{2}} + x^{\frac{7}{2}} + \sqrt{5} \Rightarrow$

$$f'(x) = 4 \cdot \left(-\frac{5}{2}\right)x^{-\frac{7}{2}} - \left(-\frac{1}{2}\right)x^{-\frac{3}{2}} + \frac{7}{2}x^{\frac{5}{2}} + 0 = -\frac{10}{x^3\sqrt{x}} + \frac{1}{2x\sqrt{x}} + \frac{7}{2}x^2\sqrt{x}$$

d. $\quad f(x) = \frac{1}{x} + \frac{1}{\pi^3} = x^{-1} + \frac{1}{\pi^3} \Rightarrow f(x) = -1 \cdot x^{-2} + 0 = -\frac{1}{x^2}$

18.

a. $\quad f(x) = -4\tan x + \sin x + 2x \Rightarrow f(x) = -\frac{4}{\cos^2 x} + \cos x + 2$

b. $\quad g(x) = \tan x - \frac{1}{\cos x} \Rightarrow$

$$g'(x) = \frac{1}{\cos^2 x} - \frac{\cos x \cdot 0 - 1 \cdot (-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} = \frac{1 - \sin x}{\cos^2 x}$$

c. $\quad f(t) = \sin t \tan t \Rightarrow f'(t) = \cos t \tan t + \sin t \cdot \frac{1}{\cos^2 t} = \sin t + \frac{\sin t}{\cos^2 t}$

d. $\quad h(t) = x\sin t + x^2\sqrt{t} \Rightarrow h(t) = x\cos t + x^2 \cdot \frac{1}{2\sqrt{t}} = x\cos t + \frac{x^2}{2\sqrt{t}}$

19.

a. $\quad f(x) = \frac{3}{\sqrt{x}} + \frac{x}{\sqrt{3}} = 3x^{-\frac{1}{2}} + \frac{1}{\sqrt{3}}x \Rightarrow$

$$f'(x) = 3 \cdot \left(-\frac{1}{2}\right)x^{-\frac{3}{2}} + \frac{1}{\sqrt{3}} = -\frac{3}{2x\sqrt{x}} + \frac{1}{3}\sqrt{3}$$

b. $\quad g(y) = \frac{3y^4}{4} + \frac{2}{5y^5} = \frac{3}{4}y^4 + \frac{2}{5}y^{-5} \Rightarrow$

$$g(y) = \frac{3}{4} \cdot 4y^3 + \frac{2}{5} \cdot (-5)y^{-6} = 3y^3 - \frac{2}{y^6}$$

c. $\quad h(t) = (3t^3 - 2t + 5)(4 - 7t) \Rightarrow$

$$h'(t) = (9t^2 - 2)(4 - 7t) + (3t^3 - 2t + 5) \cdot (-7)$$
$$= (9t^2 - 2)(4 - 7t) - 7(3t^3 - 2t + 5)$$

d. $k(t) = \dfrac{t^2 - 1}{t^2 + 1} \Rightarrow$

$$k'(t) = \frac{(t^2 + 1) \cdot (2t) - (t^2 - 1) \cdot (2t)}{(t^2 + 1)^2} = \frac{2t^3 + 2t - 2t^3 + 2t}{(t^2 + 1)^2} = \frac{4t}{(t^2 + 1)^2}$$

20.

a. $h(t) = \dfrac{t^2}{\tan t} \Rightarrow$

$$h'(t) = \frac{(\tan t) \cdot 2t - t^2 \cdot \dfrac{1}{\cos^2 t}}{(\tan t)^2} = \frac{2t \cdot \tan t \cdot \cos^2 t - t^2}{\cos^2 t \cdot \tan^2 t} = \frac{2t \cdot \sin t \cdot \cos t - t^2}{\sin^2 t}$$

b.

$$g(t) = \frac{t + \tan t}{\cos t} \Rightarrow$$

$$g'(t) = \frac{(\cos t)\left(1 + \dfrac{1}{\cos^2 t}\right) - (t + \tan t)(-\sin t)}{(\cos t)^2}$$

$$= \frac{\cos t + \dfrac{1}{\cos t} + t \sin t + \tan t \cdot \sin t}{\cos^2 t}$$

$$= \frac{\cos^2 t + 1 + t \sin t \cdot \cos t + \cos t \tan t \sin t}{\cos^3 t}$$

$$= \frac{\cos^2 t + 1 + t \sin t \cdot \cos t + \sin^2 t}{\cos^3 t} = \frac{2 + t \sin t \cdot \cos t}{\cos^3 t}$$

c. $f(x) = \sqrt[3]{x}\sqrt{x} - x^2\sqrt{3} = x^{\frac{1}{3}}x^{\frac{1}{2}} - x^2\sqrt{3} = x^{\frac{5}{6}} - \sqrt{3} \cdot x^2 \Rightarrow$

$$f'(x) = \frac{5}{6}x^{-\frac{1}{6}} - \sqrt{3} \cdot 2x = \frac{5}{6\sqrt[6]{x}} - 2\sqrt{3} \cdot x$$

d. $h(x) = x^2 \sin x \tan x \Rightarrow$

$$h'(x) = 2x \sin x \tan x + x^2 \cos x \tan x + x^2 \sin x \cdot \frac{1}{\cos^2 x}$$

$$= 2x \sin x \tan x + x^2 \sin x + \frac{x^2 \sin x}{\cos^2 x}$$

21.

Uit $f(x) = \frac{1}{3}x^2 + 2x - 1$ volgt: $f'(x) = \frac{2}{3}x + 2$.

a. Het bedoelde punt is $\left(1, f(1)\right) = \left(1, \frac{4}{3}\right)$.

De richtingscoëfficiënt van de raaklijn in dit punt is $f'(1) = 2\frac{2}{3} = \frac{8}{3}$.

De vergelijking van de raaklijn is dan $y = \frac{8}{3}x + b$. Deze lijn gaat door het punt $\left(1, \frac{4}{3}\right)$.

Invullen van dit punt geeft: $\frac{4}{3} = \frac{8}{3} \cdot 1 + b \Rightarrow b = -\frac{4}{3}$.

De vergelijking van de gevraagde raaklijn is daarmee $y = \frac{8}{3}x - \frac{4}{3}$.

b. Het bedoelde punt is $\left(0, f(0)\right) = \left(0, -1\right)$.

De richtingscoëfficiënt van de raaklijn in dit punt is $f'(0) = 2$.

De vergelijking van de raaklijn is dan $y = 2x + b$. Deze lijn gaat door het punt $\left(0, -1\right)$.

Invullen van dit punt geeft: $-1 = 2 \cdot 0 + b \Rightarrow b = -1$.

De vergelijking van de gevraagde raaklijn is daarmee $y = 2x - 1$.

c. Een horizontale raaklijn heeft richtingscoëfficiënt 0, dus voor het bedoelde punt geldt:

$f'(x) = 0$ en dus $\frac{2}{3}x + 2 = 0 \Rightarrow x = -3$

Het bedoelde punt is dus $\left(-3, f(-3)\right) = \left(-3, -4\right)$

De vergelijking van de raaklijn is dan $y = 0x + b$, dus $y = b$. Deze lijn gaat door het punt $\left(-3, -4\right)$. Invullen van dit punt geeft: $-4 = b \Rightarrow b = -4$.

De vergelijking van de gevraagde horizontale raaklijn is dus $y = -4$.

22.

a. $f(x) = \left(2x^2 - 3x + 5\right)^7 \Rightarrow f''(x) = 7 \cdot \left(2x^2 - 3x + 5\right)^6 \cdot \left(4x - 3\right)$

b. $f(t) = \left(t^4 + 3t^2 - 1\right)^{-2} \Rightarrow$

$f(t) = -2 \cdot \left(t^4 + 3t^2 - 1\right)^{-3} \cdot \left(4t^3 + 6t\right) = -4t\left(2t^2 + 3\right)\left(t^4 + 3t^2 - 1\right)^{-3}$

c. $g(t) = \left(t^3 + 2\left(t^2 - 4\right)^{-3}\right)^5 \Rightarrow$

$g'(t) = 5\left(t^3 + 2\left(t^2 - 4\right)^{-3}\right)^4 \cdot \left(3t^2 + 2 \cdot (-3)\left(t^2 - 4\right)^{-4} \cdot 2t\right)$

$= 5\left(t^3 + 2\left(t^2 - 4\right)^{-3}\right)^4 \cdot \left(3t^2 - 12t\left(t^2 - 4\right)^{-4}\right)$

d. $f(x) = \left(x^5 - 3x\right)^5 \cdot \left(1 + 3x^3\right)^3 \Rightarrow$

$f'(x) = 5\left(x^5 - 3x\right)^4\left(5x^4 - 3\right)\left(1 + 3x^3\right)^3 + \left(x^5 - 3x\right)^5 \cdot 3\left(1 + 3x^3\right)^2 \cdot 9x^2$

$= \left(x^5 - 3x\right)^4\left(1 + 3x^3\right)^2\left(5\left(5x^4 - 3\right)\left(1 + 3x^3\right) + \left(x^5 - 3x\right) \cdot 3 \cdot 9x^2\right)$

$= \left(x^5 - 3x\right)^4\left(1 + 3x^3\right)^2\left(25x^4 + 75x^7 - 15 - 45x^3 + 27x^7 - 81x^3\right)$

$= \left(x^5 - 3x\right)^4\left(1 + 3x^3\right)^2\left(102x^7 + 25x^4 - 126x^3 - 15\right)$

23.

a.

$$h(u) = \frac{3u^2 + 2u + 1}{5u - 4} \Rightarrow$$

$$h'(u) = \frac{(5u - 4)(6u + 2) - (3u^2 + 2u + 1) \cdot 5}{(5u - 4)^2}$$

$$= \frac{(30u^2 + 10u - 24u - 8) - (15u^2 + 10u + 5)}{(5u - 4)^2} = \frac{15u^2 - 24u - 13}{(5u - 4)^2}$$

b.

$$k(t) = \frac{t^2 + 1}{(t^2 - 1)^2} \Rightarrow$$

$$k'(t) = \frac{(t^2 - 1)^2 \cdot 2t - (t^2 + 1) \cdot 2(t^2 - 1) \cdot 2t}{(t^2 - 1)^4}$$

$$= \frac{2t(t^2 - 1)\left((t^2 - 1) - 2(t^2 + 1)\right)}{(t^2 - 1)^4} = \frac{2t(-t^2 - 3)}{(t^2 - 1)^3} = \frac{-2t(t^2 + 3)}{(t^2 - 1)^3}$$

of

$$k(t) = \frac{t^2 + 1}{(t^2 - 1)^2} = (t^2 + 1)(t^2 - 1)^{-2} \Rightarrow$$

$$k'(t) = 2t(t^2 - 1)^{-2} + (t^2 + 1)(-2)(t^2 - 1)^{-3} \cdot 2t$$

$$= \frac{2t}{(t^2 - 1)^2} - \frac{4t(t^2 + 1)}{(t^2 - 1)^3} = \frac{2t(t^2 - 1) - 4t(t^2 + 1)}{(t^2 - 1)^3} =$$

$$= \frac{2t(t^2 - 1 - 2t^2 - 2)}{(t^2 - 1)^3} = \frac{2t(-t^2 - 3)}{(t^2 - 1)^3} = \frac{-2t(t^2 + 3)}{(t^2 - 1)^3}$$

c.

$$g(t) = \left(\frac{t^2 + 6^3}{5 - 3t}\right)^{-5} \Rightarrow$$

$$g'(t) = -5 \cdot \left(\frac{t^2 + 6^3}{5 - 3t}\right)^{-6} \cdot \frac{(5 - 3t)(2t + 0) - (t^2 + 6^3) \cdot (-3)}{(5 - 3t)^2}$$

$$= -5 \cdot \left(\frac{5 - 3t}{t^2 + 6^3}\right)^6 \cdot \frac{10t - 6t^2 + 3t^2 + 3 \cdot 6^3}{(5 - 3t)^2}$$

$$= \frac{-5 \cdot (5 - 3t)^4 \left(-3t^2 + 10t + 648\right)}{(t^2 + 216)^6} = \frac{5(5 - 3t)^4 \left(3t^2 - 10t - 648\right)}{(t^2 + 216)^6}$$

of

$$g(t) = \left(\frac{t^2 + 6^3}{5 - 3t}\right)^{-5} = \left(\frac{5 - 3t}{t^2 + 6^3}\right)^5 \Rightarrow$$

$$g'(t) = 5 \cdot \left(\frac{5 - 3t}{t^2 + 6^3}\right)^4 \cdot \frac{\left(t^2 + 6^3\right) \cdot (-3) - (5 - 3t)(2t + 0)}{\left(t^2 + 6^3\right)^2}$$

$$= 5 \cdot \left(\frac{5 - 3t}{t^2 + 6^3}\right)^4 \cdot \frac{-3t^2 - 3 \cdot 6^3 - 10t + 6t^2}{\left(t^2 + 6^3\right)^2} = \frac{5 \cdot (5 - 3t)^4 \left(3t^2 - 10t - 648\right)}{\left(t^2 + 216\right)^6}$$

d. $\quad f(x) = \dfrac{\left(x^2 + 1\right)^3}{(5 - 3x)^4} \Rightarrow$

$$f'(x) = \frac{(5 - 3x)^4 \cdot 3\left(x^2 + 1\right)^2 \cdot 2x - \left(x^2 + 1\right)^3 \cdot 4 \cdot (5 - 3x)^3 \cdot (-3)}{(5 - 3x)^8}$$

$$= \frac{3 \cdot 2 \cdot (5 - 3x)^3 \left(x^2 + 1\right)^2 \left((5 - 3x) \cdot x - \left(x^2 + 1\right) \cdot 2 \cdot (-1)\right)}{(5 - 3x)^8}$$

$$= \frac{6\left(x^2 + 1\right)^2 \left(5x - 3x^2 + 2x^2 + 2\right)}{(5 - 3x)^5} = \frac{-6\left(x^2 + 1\right)^2 \left(x^2 - 5x - 2\right)}{(5 - 3x)^5}$$

of

$$f(x) = \frac{\left(x^2 + 1\right)^3}{(5 - 3x)^4} = \left(x^2 + 1\right)^3 \cdot (5 - 3x)^{-4} \Rightarrow$$

$$f'(x) = 3\left(x^2 + 1\right)^2 \cdot 2x \cdot (5 - 3x)^{-4} + \left(x^2 + 1\right)^3 \cdot (-4) \cdot (5 - 3x)^{-5} \cdot (-3)$$

$$= \frac{6x\left(x^2 + 1\right)^2}{(5 - 3x)^4} + \frac{12\left(x^2 + 1\right)^3}{(5 - 3x)^5} = \frac{6x\left(x^2 + 1\right)^2 (5 - 3x) + 12\left(x^2 + 1\right)^3}{(5 - 3x)^5}$$

$$= \frac{6\left(x^2 + 1\right)^2 \cdot \left(x(5 - 3x) + 2\left(x^2 + 1\right)\right)}{(5 - 3x)^5} = \frac{6\left(x^2 + 1\right)^2 \cdot \left(-x^2 + 5x + 2\right)}{(5 - 3x)^5}$$

$$= \frac{-6\left(x^2 + 1\right)^2 \left(x^2 - 5x - 2\right)}{(5 - 3x)^5}$$

24.

a. $\quad f(x) = \dfrac{1}{\sqrt[5]{x^4 + 2}} = \left(x^4 + 2\right)^{-\frac{1}{5}} \Rightarrow$

$$f'(x) = -\frac{1}{5}\left(x^4 + 2\right)^{-1\frac{1}{5}} \cdot 4x^3 = -\frac{4x^3}{5\left(x^4 + 2\right)\sqrt[5]{x^4 + 2}}$$

b. $\quad f(x) = \dfrac{1}{\sqrt{3x - 5}} = (3x - 5)^{-\frac{1}{2}} \Rightarrow f'(x) = -\frac{1}{2}(3x - 5)^{-1\frac{1}{2}} \cdot 3 = -\dfrac{3}{2(3x - 5)\sqrt{3x - 5}}$

c. $\quad g(t) = \dfrac{t}{\sqrt{6t^4 + 5}} = t \cdot \left(6t^4 + 5\right)^{-\frac{1}{2}} \Rightarrow$

$$g(t) = \left(6t^4 + 5\right)^{-\frac{1}{2}} + t \cdot -\frac{1}{2}\left(6t^4 + 5\right)^{-1\frac{1}{2}} \cdot 24t^3 = \frac{1}{\sqrt{6t^4 + 5}} - \frac{12t^4}{\left(6t^4 + 5\right)\sqrt{6t^4 + 5}}$$

$$= \frac{6t^4 + 5}{\left(6t^4 + 5\right)\sqrt{6t^4 + 5}} - \frac{12t^4}{\left(6t^4 + 5\right)\sqrt{6t^4 + 5}} = \frac{-6t^4 + 5}{\left(6t^4 + 5\right)\sqrt{6t^4 + 5}}$$

d. $\quad g(t) = \sqrt{2t - \sqrt{3t}} = \left(2t - \sqrt{3} \cdot t^{\frac{1}{2}}\right)^{\frac{1}{2}} \Rightarrow$

$$g(t) = \frac{1}{2}\left(2t - \sqrt{3} \cdot t^{\frac{1}{2}}\right)^{-\frac{1}{2}} \cdot \left(2 - \sqrt{3} \cdot \frac{1}{2}t^{-\frac{1}{2}}\right) = \frac{2 - \dfrac{\sqrt{3}}{2\sqrt{t}}}{2\sqrt{2t - \sqrt{3t}}} = \frac{4\sqrt{t} - \sqrt{3}}{4\sqrt{t}\sqrt{2t - \sqrt{3t}}}$$

25.

a. $\quad f(x) = \dfrac{\tan^4 x}{\cos^3 x} = \dfrac{\left(\tan x\right)^4}{\left(\cos x\right)^3} \Rightarrow$

$$f'(x) = \frac{\left(\cos x\right)^3 \cdot 4\left(\tan x\right)^3 \cdot \dfrac{1}{\left(\cos x\right)^2} - \left(\tan x\right)^4 \cdot 3\left(\cos x\right)^2\left(-\sin x\right)}{\left(\cos x\right)^6}$$

$$= \frac{4\left(\cos x\right)\left(\tan x\right)^3 + 3\left(\tan x\right)^4\left(\cos x\right)^2\left(\sin x\right)}{\left(\cos x\right)^6}$$

$$= \frac{4\left(\tan x\right)^3 + 3\left(\tan x\right)^4\left(\cos x\right)\left(\sin x\right)}{\left(\cos x\right)^5}$$

$$= \frac{4\tan^3 x + 3\tan^4 x \cdot \cos x \cdot \sin x}{\left(\cos x\right)^5} = \frac{4\dfrac{\sin^3 x}{\cos^3 x} + 3\dfrac{\sin^4 x}{\cos^4 x} \cdot \cos x \cdot \sin x}{\left(\cos x\right)^5}$$

$$= \frac{4\dfrac{\sin^3 x}{\cos^3 x} + 3\dfrac{\sin^5 x}{\cos^3 x}}{\left(\cos x\right)^5} = \frac{4\sin^3 x + 3\sin^5 x}{\left(\cos x\right)^8}$$

b. $\quad f(x) = \sqrt{\dfrac{\sin^2 x}{\cos^3 x}} = \left(\dfrac{\sin^2 x}{\cos^3 x}\right)^{\frac{1}{2}} \Rightarrow$

$$f'(x) = \frac{1}{2}\left(\frac{\sin^2 x}{\cos^3 x}\right)^{-\frac{1}{2}} \cdot \frac{\cos^3 x \cdot 2\sin x \cdot \cos x - \sin^2 x \cdot 3\cos^2 x \cdot \left(-\sin x\right)}{\cos^6 x}$$

$$= \left(\frac{\cos^3 x}{\sin^2 x}\right)^{\frac{1}{2}} \cdot \frac{2\cos^2 x \cdot \sin x + 3\sin^3 x}{2\cos^4 x}$$

$$= \sqrt{\frac{\cos^3 x}{\sin^2 x}} \cdot \frac{2\cos^2 x \cdot \sin x + 3\sin^3 x}{2\cos^4 x}$$

c. $\quad f(t) = \sin^3\left(2t\right) + \cos^3\left(2t\right) \Rightarrow$

$$f'(t) = 3\sin^2(2t) \cdot \cos(2t) \cdot 2 + 3\cos^2(2t) \cdot \left(-\sin(2t)\right) \cdot 2$$

$$= 6\sin(2t)\cos(2t)\left(\sin(2t) - \cos(2t)\right) = 3\sin(4t)\left(\sin(2t) - \cos(2t)\right)$$

d. $\quad f(t) = \tan^4(2t) + \dfrac{1}{\cos(2t)} = \tan^4(2t) + \left(\cos(2t)\right)^{-1} \Rightarrow$

$$f'(t) = 4\tan^3(2t) \cdot \dfrac{1}{\cos^2(2t)} \cdot 2 + (-1)\left(\cos(2t)\right)^{-2} \cdot \left(-\sin(2t)\right) \cdot 2$$

$$= \dfrac{8\tan^3(2t)}{\cos^2(2t)} + \dfrac{2\sin(2t)}{\cos^2(2t)} = \dfrac{8\tan^3(2t) + 2\sin(2t)}{\cos^2(2t)}$$

26.

a. $\quad f(x) = e^{-x^3} \Rightarrow f'(x) = e^{-x^3} \cdot \left(-3x^2\right) = -3x^2 e^{-x^3}$

b. $\quad f(x) = xe^{\sin(2x)} \Rightarrow$

$$f'(x) = 1 \cdot e^{\sin(2x)} + xe^{\sin(2x)} \cdot \cos(2x) \cdot 2 = \left(1 + 2x\cos(2x)\right) \cdot e^{\sin(2x)}$$

c. $\quad f(x) = \dfrac{e^{x+\sin(2x)}}{\tan(3x)} \Rightarrow$

$$f'(x) = \dfrac{\tan(3x) \cdot e^{x+\sin(2x)} \cdot \left(1 + \cos(2x) \cdot 2\right) - e^{x+\sin(2x)} \cdot \dfrac{1}{\cos^2(3x)} \cdot 3}{\left(\tan(3x)\right)^2}$$

$$= \dfrac{\cos^2(3x) \cdot \tan(3x) \cdot e^{x+\sin(2x)} \cdot \left(1 + \cos(2x) \cdot 2\right) - 3e^{x+\sin(2x)}}{\cos^2(3x) \cdot \tan^2(3x)}$$

$$= \dfrac{\left(\cos(3x) \cdot \sin(3x) \cdot \left(1 + 2\cos(2x)\right) - 3\right) \cdot e^{x+\sin(2x)}}{\sin^2(3x)}$$

$$= \dfrac{\left(\cos(3x) \cdot \sin(3x) + 2\cos(2x)\cos(3x)\sin(3x) - 3\right) \cdot e^{x+\sin(2x)}}{\sin^2(3x)}$$

d. $\quad f(x) = e^{x^3 \sin\left(2x - \frac{1}{3}\pi\right)} \Rightarrow$

$$f'(x) = e^{x^3 \sin\left(2x - \frac{1}{3}\pi\right)} \cdot \left(3x^2 \sin\left(2x - \tfrac{1}{3}\pi\right) + x^3 \cos\left(2x - \tfrac{1}{3}\pi\right) \cdot 2\right)$$

$$= x^2 \left(3\sin\left(2x - \tfrac{1}{3}\pi\right) + 2x\cos\left(2x - \tfrac{1}{3}\pi\right)\right) e^{x^3 \sin\left(2x - \frac{1}{3}\pi\right)}$$

27.

a. $\quad g(t) = \ln\left(t^3\right) \Rightarrow g'(t) = \dfrac{1}{t^3} \cdot 3t^2 = \dfrac{3}{t}$

of

$$g(t) = \ln\left(t^3\right) = 3\ln(t) \Rightarrow g'(t) = 3 \cdot \dfrac{1}{t} = \dfrac{3}{t}$$

b. $\quad g(t) = \left(\ln t\right)^3 \Rightarrow g'(t) = 3\left(\ln t\right)^2 \cdot \dfrac{1}{t} = \dfrac{3\ln^2 t}{t} = \dfrac{3}{t} \cdot \ln^2 t$

c. $\quad g(t) = \ln\left(\ln\left(\ln t\right)\right) \Rightarrow g'(t) = \dfrac{1}{\ln(\ln t)} \cdot \dfrac{1}{\ln t} \cdot \dfrac{1}{t} = \dfrac{1}{t \cdot \ln t \cdot \ln(\ln t)}$

d. $\quad g(t) = \sqrt{\ln\left(\sin\left(3t\right)\right)} \Rightarrow$

$$g'(t) = \frac{1}{2\sqrt{\ln\left(\sin\left(3t\right)\right)}} \cdot \frac{1}{\sin\left(3t\right)} \cdot \cos\left(3t\right) \cdot 3$$

$$= \frac{3\cos\left(3t\right)}{2\sin\left(3t\right)\sqrt{\ln\left(\sin\left(3t\right)\right)}} = \frac{3}{2\tan\left(3t\right)\sqrt{\ln\left(\sin\left(3t\right)\right)}}$$

28.

a. $\quad f(t) = 7^{2t^3-3t} \Rightarrow f'(t) = 7^{2t^3-3t} \cdot \ln 7 \cdot \left(6t - 3\right) = 3\ln 7 \cdot \left(2t - 1\right) \cdot 7^{2t^3-3t}$

b. $\quad f(t) = 2^{3\log\left(4t+5\right)} \Rightarrow$

$$f'(t) = 2^{3\log\left(4t+5\right)} \cdot \ln 2 \cdot \frac{1}{\left(4t + 5\right)\ln 3} \cdot 4 = \frac{4 \cdot \ln\left(2\right)}{\ln\left(3\right) \cdot \left(4t + 5\right)} \cdot 2^{3\log\left(4t+5\right)}$$

c. $\quad g(x) = \ln\left(xe^x\right) \Rightarrow g'(x) = \frac{1}{xe^x}\left(1 \cdot e^x + x \cdot e^x\right) = \frac{e^x\left(1 + x\right)}{xe^x} = \frac{1 + x}{x} = \frac{1}{x} + 1$

of

$$g(x) = \ln\left(xe^x\right) = \ln\left(x\right) + \ln\left(e^x\right) = \ln\left(x\right) + x \Rightarrow g'(x) = \frac{1}{x} + 1$$

d. $\quad h(t) = \dfrac{\ln\left(2t^3\right)}{t^6} \Rightarrow$

$$h'(t) = \frac{t^6 \cdot \left(\dfrac{1}{2t^3} \cdot 6t^2\right) - \ln\left(2t^3\right) \cdot 6t^5}{\left(t^6\right)^2} = \frac{3t^5 - \ln\left(2t^3\right) \cdot 6t^5}{t^{12}} = \frac{3 - 6\ln\left(2t^3\right)}{t^7}$$

of

$$h(t) = \frac{\ln\left(2t^3\right)}{t^6} = \frac{\ln\left(2\right) + \ln\left(t^3\right)}{t^6} = \frac{\ln\left(2\right) + 3\ln t}{t^6} \Rightarrow$$

$$h(t) = \frac{t^6 \cdot \left(0 + \dfrac{3}{t}\right) - \left(\ln\left(2\right) + 3\ln t\right) \cdot 6t^5}{t^{12}} = \frac{3t^5 - \left(\ln\left(2\right) + 3\ln t\right) \cdot 6t^5}{t^{12}}$$

$$= \frac{3 - \left(\ln\left(2\right) + 3\ln t\right) \cdot 6}{t^7} = \frac{3 - 6\ln\left(2\right) - 18\ln t}{t^7}$$

29.

a. $\quad g(t) = \arccos\left(3t\right) - \arcsin\left(3t\right) \Rightarrow$

$$g'(t) = \frac{-1}{\sqrt{1 - \left(3t\right)^2}} \cdot 3 - \frac{1}{\sqrt{1 - \left(3t\right)^2}} \cdot 3 = \frac{-3}{\sqrt{1 - 9t^2}} - \frac{3}{\sqrt{1 - 9t^2}} = \frac{-6}{\sqrt{1 - 9t^2}}$$

b. $\quad f(x) = \arccos\left(x^3\right) \Rightarrow f'(x) = \frac{-1}{\sqrt{1 - \left(x^3\right)^2}} \cdot 3x^2 = \frac{-3x^2}{\sqrt{1 - x^6}}$

c. $\quad k(t) = t\arccos\left(\sqrt{1 - t^2}\right) \Rightarrow$

$$k'(t) = 1 \cdot \arccos\left(\sqrt{1 - t^2}\right) + t \cdot \frac{1}{\sqrt{1 - \left(\sqrt{1 - t^2}\right)^2}} \cdot \frac{1}{2\sqrt{1 - t^2}} \cdot (0 - 2t)$$

$$= \arccos\left(\sqrt{1 - t^2}\right) - \frac{t^2}{\sqrt{1 - \left(1 - t^2\right)}\sqrt{1 - t^2}}$$

$$= \arccos\left(\sqrt{1 - t^2}\right) - \frac{t^2}{\sqrt{t^2}\sqrt{1 - t^2}}$$

$$= \arccos\left(\sqrt{1 - t^2}\right) - \frac{t^2}{|t|\sqrt{1 - t^2}} = \arccos\left(\sqrt{1 - t^2}\right) - \frac{|t|}{\sqrt{1 - t^2}}$$

30.

a. $g(u) = 2\arctan\left(\frac{5}{u}\right) + 5\ln\left(u^2 + 25\right) \Rightarrow$

$$g(u) = 2 \cdot \frac{1}{1 + \left(\frac{5}{u}\right)^2} \cdot \left(-\frac{5}{u^2}\right) + 5 \cdot \frac{1}{u^2 + 25} \cdot 2u$$

$$= \frac{-10}{u^2 + 25} + \frac{10u}{u^2 + 25} = \frac{10u - 10}{u^2 + 25}$$

b. $f(t) = \left(t + \arctan(3t)\right)^2 \Rightarrow$

$$f'(t) = 2 \cdot \left(t + \arctan(3t)\right) \cdot \left(1 + \frac{1}{1 + (3t)^2} \cdot 3\right)$$

$$= 2 \cdot \left(t + \arctan(3t)\right) \cdot \left(1 + \frac{3}{1 + 9t^2}\right)$$

$$= 2 \cdot \left(t + \arctan(3t)\right) \cdot \frac{9t^2 + 4}{1 + 9t^2} = \frac{18t^2 + 8}{1 + 9t^2} \cdot \left(t + \arctan(3t)\right)$$

c. $g(x) = \dfrac{x^2 + 1}{\arctan x} \Rightarrow$

$$g'(x) = \frac{\arctan x \cdot (2x + 0) - \left(x^2 + 1\right) \cdot \dfrac{1}{x^2 + 1}}{\left(\arctan x\right)^2} = \frac{2x\arctan x - 1}{\left(\arctan x\right)^2}$$

31.

a. $y = \frac{1}{4}x^8 - 5x^3 + 2x \Rightarrow$

$$dy = d\left(\frac{1}{4}x^8 - 5x^3 + 2x\right) = d\left(\frac{1}{4}x^8\right) - d\left(5x^3\right) + d(2x)$$

$$= 2x^7 dx - 15x^2 dx + 2dx = \left(2x^7 - 15x^2 + 2\right)dx \Rightarrow$$

$$dx = \frac{1}{2x^7 - 15x^2 + 2}dy$$

b. $x^4 y^2 - x^2 y^4 = 4x^3 y^3 \Rightarrow$

$$d\left(x^4y^2 - x^2y^4\right) = d\left(4x^3y^3\right) \Rightarrow$$

$$d\left(x^4y^2\right) - d\left(x^2y^4\right) = d\left(4x^3y^3\right) \Rightarrow$$

$$\left(d\left(x^4\right)y^2 + x^4d\left(y^2\right)\right) - \left(d\left(x^2\right)y^4 + x^2d\left(y^4\right)\right) = d\left(4x^3\right)y^3 + 4x^3d\left(y^3\right) \Rightarrow$$

$$\left(4x^3y^2dx + 2x^4ydy\right) - \left(2xy^4dx + 4x^2y^3dy\right) = 12x^2y^3dx + 12x^3y^2dy \Rightarrow$$

$$4x^3y^2dx - 2xy^4dx - 12x^2y^3dx = 12x^3y^2dy - 2x^4ydy + 4x^2y^3dy \Rightarrow$$

$$2xy^2\left(2x^2 - y^2 - 6xy\right)dx = 2x^2y\left(6xy - x^2 + 2y^2\right)dy \Rightarrow$$

$$dx = \frac{2x^2y\left(6xy - x^2 + 2y^2\right)}{2xy^2\left(2x^2 - y^2 - 6xy\right)}dy \Rightarrow dx = -\frac{x\left(x^2 - 6xy - 2y^2\right)}{y\left(2x^2 - 6xy - y^2\right)}dy$$

c. $\quad z^2 \cdot t = \sin\left(z^2 + t\right) \Rightarrow$

$$d\left(z^2 \cdot t\right) = d\left(\sin\left(z^2 + t\right)\right) \Rightarrow$$

$$d\left(z^2\right) \cdot t + z^2 \cdot d(t) = \cos\left(z^2 + t\right) \cdot d\left(z^2 + t\right) \Rightarrow$$

$$2z\,dz \cdot t + z^2 \cdot dt = \cos\left(z^2 + t\right) \cdot \left(2z\,dz + dt\right) \Rightarrow$$

$$2zt\,dz - 2z\cos\left(z^2 + t\right)dz = \cos\left(z^2 + t\right)dt - z^2dt \Rightarrow$$

$$\left(2zt - 2z\cos\left(z^2 + t\right)\right)dz = \left(\cos\left(z^2 + t\right) - z^2\right)dt \Rightarrow$$

$$dz = \frac{\cos\left(z^2 + t\right) - z^2}{2zt - 2z\cos\left(z^2 + t\right)}dt$$

d. $\quad 5x^3 \cdot \log y + 3y^5 \cdot \log x = x^2\sqrt{x} \Rightarrow$

$$d\left(5x^3\right) \cdot \log y + 5x^3 \cdot d\left(\log y\right) + d\left(3y^5\right) \cdot \log x + 3y^5 \cdot d\left(\log x\right) = d\left(x^{\frac{5}{2}}\right) \Rightarrow$$

$$15x^2dx \cdot \log y + 5x^3 \cdot \frac{1}{y\ln 10}dy + 15y^2dy \cdot \log x + 3y^5\frac{1}{x\ln 10}dx = \frac{5}{2}x^{\frac{3}{2}}dx \Rightarrow$$

$$\left(\frac{5x^3}{y\ln 10} + 15y^2\log x\right)dy = \left(\frac{5}{2}x^{\frac{3}{2}} - 15x^2\log y - \frac{3y^5}{x\ln 10}\right)dx \Rightarrow$$

$$[\text{linker- en rechterlid met } 2xy\ln 10 \text{ vermeningvuldigen}]$$

$$\left(10x^4 + 30\ln 10 \cdot xy^3\log x\right)dy = \left(5\ln 10 \cdot xyx\sqrt{x} - 30\ln 10 \cdot x^3y\log y - 6y^6\right)dx \Rightarrow$$

$$dy = \frac{10x^4 + 30 \cdot \ln 10 \cdot xy^3\log x}{5y \cdot \ln 10 \cdot x\sqrt{x} - 30 \cdot \ln 10 \cdot x^3y \cdot \log y - 6y^6}dx$$

32.

a. $\quad x^3 + 3x^2y + y^3 = 5 \Rightarrow$

$$d\left(x^3 + 3x^2y + y^3\right) = d(5) \Rightarrow$$

$$3x^2dx + 6xdx \cdot y + 3x^2dy + 3y^2dy = 0 \Rightarrow$$

$$3\left(x^2 + y^2\right)dy = -3x(x + 2y)dx \Rightarrow$$

$$\frac{dy}{dx} = \frac{-3x(x+2y)}{3(x^2+y^2)} = -\frac{x(x+2y)}{x^2+y^2}$$

b. $\dfrac{x^2}{y} - \dfrac{y}{x^2} = 1 \Rightarrow$

$$d\left(\frac{x^2}{y}\right) - d\left(\frac{y}{x^2}\right) = d(1) \Rightarrow$$

$$\frac{y \cdot d(x^2) - x^2 \cdot d(y)}{y^2} - \frac{x^2 \cdot d(y) - y \cdot d(x^2)}{x^4} = 0 \Rightarrow$$

$$\frac{2xydx - x^2dy}{y^2} - \frac{x^2dy - 2xydx}{x^4} = 0 \Rightarrow$$

$$\frac{x^4(2xydx - x^2dy) - y^2(x^2dy - 2xydx)}{x^4y^2} = 0 \Rightarrow$$

$$2x^5ydx - x^6dy - x^2y^2dy + 2xy^3dx = 0 \Rightarrow$$

$$-x^6dy - x^2y^2dy = -2x^5ydx - 2xy^3dx \Rightarrow$$

$$x^2(x^4 + y^2)dy = 2xy(x^4 + y^2)dx \Rightarrow$$

$$\frac{dy}{dx} = \frac{2xy(x^4 + y^2)}{x^2(x^4 + y^2)} = 2\frac{y}{x}$$

33.

a. $x^2 - y^2 = 6xy - 10 \Rightarrow$

$$d(x^2 - y^2) = d(6xy - 10) \Rightarrow$$

$$2xdx - 2ydy = 6dx \cdot y + 6xdy - 0 \Rightarrow$$

$$-2ydy - 6xdy = -2xdx + 6ydx \Rightarrow$$

$$ydy + 3xdy = xdx - 3ydx \Rightarrow$$

$$(y + 3x)dy = (x - 3y)dx \Rightarrow$$

$$\frac{dy}{dx} = \frac{x - 3y}{3x + y}$$

b.   We controleren of $P(-3,19)$ op de kromme K ligt.

We vullen $x = -3$ en $y = 19$ in de kromme $K : x^2 - y^2 = 6xy - 10$ in:

Geldt $(-3)^2 - 19^2 = 6 \cdot (-3) \cdot 19 - 10$?

Oftewel geldt $9 - 361 = -342 - 10$?

Ja, dit klopt, want $-352 = -352$.

De richtingscoëfficiënt a van de raaklijn m in het punt $P(-3,19)$ aan de kromme K

is: $a = \left[\dfrac{dy}{dx}\right]_{(x,y)=(-3,19)} = \left[\dfrac{x - 3y}{3x + y}\right]_{(x,y)=(-3,19)} = \dfrac{-3 - 3 \cdot 19}{3 \cdot (-3) + 19} = \dfrac{-60}{10} = -6.$

De vergelijking van de raaklijn m is dus: $y = -6x + b$.

Deze lijn gaat door $P(-3,19)$.

Dus geldt: $19 = -6 \cdot (-3) + b \Rightarrow 19 = 18 + b \Rightarrow b = 1$.

De vergelijking van de gevraagde raaklijn *m* is dus: $y = -6x + 1$

c. Een raaklijn loopt horizontaal als de richtingscoëfficiënt gelijk is aan 0.

Dus moet gelden: $\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{x - 3y}{3x + y} = 0 \Rightarrow x - 3y = 0 \Rightarrow x = 3y$

Het punt moet op de kromme K liggen; we vullen $x = 3y$ in de vergelijking van K in:

$(3y)^2 - y^2 = 6(3y)y - 10 \Rightarrow 9y^2 - y^2 = 18y^2 - 10 \Rightarrow$

$10y^2 = 10 \Rightarrow y^2 = 1 \Rightarrow y = 1 \vee y = -1$

Samen met $x = 3y$ geeft dit de punten: $(3,1)$ en $(-3,-1)$

d. Een verticale lijn loop 'oneindig' steil, de richtingscoëfficiënt is dus 'oneindig' (of '-oneindig').

Dan moet gelden: $\dfrac{dy}{dx} = \pm\infty$, oftewel $\dfrac{dx}{dy} = 0$. Dit geeft:

$\dfrac{dx}{dy} = 0 \Rightarrow \dfrac{3x + y}{x - 3y} = 0 \Rightarrow 3x + y = 0 \Rightarrow y = -3x$

Het punt moet op de K liggen; we vullen $y = -3x$ in de vergelijking van K in:

$x^2 - y^2 = 6xy - 10 \Rightarrow x^2 - (-3x)^2 = 6x(-3x) - 10 \Rightarrow$

$x^2 - 9x^2 = -18x^2 - 10 \Rightarrow 10x^2 = -10 \Rightarrow x^2 = -1$

Deze vergelijking heeft dus geen oplossingen. Er zijn dusgeen punten waarin de raaklijn verticaal loopt.

34.

a.

$$\lim_{u \to 0} \frac{2^{3u} - 2^{-u}}{\sin(5u)} = \left(\frac{1-1}{0}\right) \underset{\langle \text{l' Hôpital} \rangle}{=} \lim_{u \to 0} \frac{\dfrac{d}{du}\left(2^{3u} - 2^{-u}\right)}{\dfrac{d}{du}\left(\sin(5u)\right)}$$

$$= \lim_{u \to 0} \frac{2^{3u} \cdot \ln 2 \cdot 3 - 2^{-u} \cdot \ln 2 \cdot (-1)}{\cos(5u) \cdot 5}$$

$$= \lim_{u \to 0} \frac{3\ln 2 \cdot 2^{3u} + \ln 2 \cdot 2^{-u}}{5\cos(5u)} = \frac{3\ln 2 + \ln 2}{5} = \frac{4\ln 2}{5} = \frac{4}{5}\ln 2$$

b.

$$\lim_{x \to 0} \frac{\cos(2x) - 1}{x^2} = \left(\frac{1-1}{0} = \frac{0}{0}\right) \underset{\langle \text{l'Hôpital} \rangle}{=} \lim_{x \to 0} \frac{\dfrac{d}{dx}\left(\cos(2x) - 1\right)}{\dfrac{d}{dx}\left(x^2\right)} = \lim_{x \to 0} \frac{-2\sin(2x)}{2x}$$

$$= \lim_{x \to 0} \frac{-\sin(2x)}{x} = \left(\frac{0}{0}\right) \underset{\langle \text{l'Hôpital} \rangle}{=} \lim_{x \to 0} \frac{\dfrac{d}{dx}\left(-\sin(2x)\right)}{\dfrac{d}{dx}(x)}$$

$$= \lim_{x \to 0} \frac{-2\cos(2x)}{1} = -2$$

c.

$$\lim_{t \to -\infty} \left(t^2 \cdot e^{3t} \right) = \left(-\infty \cdot 0 \right) = \lim_{t \to -\infty} \frac{t^2}{e^{-3t}} = \left(\frac{\infty}{\infty} \right) \underset{\langle \text{l'Hôpital} \rangle}{=} \lim_{t \to -\infty} \frac{\frac{d}{dt}\left(t^2 \right)}{\frac{d}{dt}\left(e^{-3t} \right)}$$

$$= \lim_{t \to -\infty} \frac{2t}{-3e^{-3t}} = \left(\frac{\infty}{-\infty} \right) \underset{\langle \text{l'Hôpital} \rangle}{=} \lim_{t \to -\infty} \frac{\frac{d}{dt}(2t)}{\frac{d}{dt}\left(-3e^{-3t} \right)}$$

$$= \lim_{t \to -\infty} \frac{2}{9e^{-3t}} = \left(\frac{2}{\infty} \right) = 0$$

35.

a. $$\lim_{x \to 2} \frac{x^6 - 64}{x - 2} = \left(\frac{64 - 64}{2 - 2} = \frac{0}{0} \right) \underset{\langle \text{l'Hôpital} \rangle}{=} \lim_{x \to 2} \frac{\frac{d}{dx}\left(x^6 - 64 \right)}{\frac{d}{dx}(x - 2)} = \lim_{x \to 2} \frac{6x^5}{1} = 192$$

b.

$$\lim_{v \to -\infty} \frac{3v^4 - 4v + 5}{3v^3 - 2v^5} = \left(\frac{\pm\infty}{\pm\infty} \right) = \lim_{v \to -\infty} \frac{3\dfrac{v^4}{v^5} - 4\dfrac{v}{v^5} + \dfrac{5}{v^5}}{3\dfrac{v^3}{v^5} - 2\dfrac{v^5}{v^5}}$$

$$= \lim_{v \to -\infty} \frac{3\dfrac{1}{v} - 4\dfrac{1}{v^4} + \dfrac{5}{v^5}}{3\dfrac{1}{v^2} - 2} = \frac{0 - 0 + 0}{0 - 2} = 0$$

Berekening kan ook met het meerder keren toepassen van de regel van l´Hôpital.

c. $$\lim_{t \to \infty} \frac{2 + t^2\sqrt{t}}{1 + 3t^2} = \left(\frac{\pm\infty}{\pm\infty} \right) = \lim_{t \to \infty} \frac{\dfrac{2}{t^2} + \dfrac{t^2\sqrt{t}}{t^2}}{\dfrac{1}{t^2} + \dfrac{3t^2}{t^2}} = \lim_{t \to \infty} \frac{\dfrac{2}{t^2} + \sqrt{t}}{\dfrac{1}{t^2} + 3} = \left(\frac{0 + \infty}{0 + 3} \right) = \infty$$

Berekening kan ook met het meerder keren toepassen van de regel van l´Hôpital.

36.

a. $$\lim_{x \to 2} \frac{x^2\sqrt{x + 7} - ax + 3}{x - 2} = \left(\frac{4\sqrt{9} - 2a + 3}{2 - 2} = \frac{15 - 2a}{0} \right)$$

Deze limiet kan alleen als reëel getal bestaan als naast de noemer ook de teller gelijk is aan 0.

Dat betekent dat $15 - 2a = 0$, dus: $a = \dfrac{15}{2} = 7\dfrac{1}{2}$

b.

$$\lim_{x \to 2} \frac{x^2\sqrt{x+7} - ax + 3}{x - 2} = \left\langle \text{vul in: } a = \frac{15}{2} \right\rangle = \lim_{x \to 2} \frac{x^2\sqrt{x+7} - \frac{15}{2}x + 3}{x - 2}$$

$$= \left(\frac{0}{0}\right)_{\langle \text{l'Hôpital} \rangle} = \lim_{x \to 2} \frac{\dfrac{d}{dx}\left(x^2\sqrt{x+7} - \frac{15}{2}x + 3\right)}{\dfrac{d}{dx}(x - 2)}$$

$$= \lim_{x \to 2} \frac{2x\sqrt{x+7} + x^2 \dfrac{1}{2\sqrt{x+7}} - \dfrac{15}{2}}{1}$$

$$= \lim_{x \to 2} 2x\sqrt{x+7} + x^2 \frac{1}{2\sqrt{x+7}} - \frac{15}{2}$$

$$= 4\sqrt{9} + 4 \cdot \frac{1}{2\sqrt{9}} - \frac{15}{2} = 12 + \frac{2}{3} - \frac{15}{2} = \frac{31}{6} = 5\frac{1}{6}$$