drs. J.H. Blankespoor
drs. C. de Joode
ir. A. Sluijter

Toegepaste Wiskunde
voor het hoger beroepsonderwijs

Deel 1
Vijfde, herziene druk

Uitwerking herhalingsopgaven hoofdstuk 6

Uitwerking herhalingsopgaven hoofdstuk 6, paragraaf 6.8

Opgave 1

a

$$\begin{aligned}\int 3x\sqrt{1-2x^2}\,\mathrm{d}x &= -\tfrac{3}{4}\int \sqrt{1-2x^2}\left(1-2x^2\right)'\,\mathrm{d}x\\ &= -\tfrac{3}{4}\int \left(1-2x^2\right)^{\frac{1}{2}}\,\mathrm{d}\left(1-2x^2\right)\\ &= -\tfrac{3}{4}\tfrac{2}{3}\left(1-2x^2\right)^{\frac{3}{2}}+C\\ &= -\tfrac{1}{2}\left(1-2x^2\right)\sqrt{1-2x^2}+C\end{aligned}$$

b

$$\begin{aligned}\int \frac{8t^2}{\left(t^3+2\right)^3}\,\mathrm{d}t &= \tfrac{8}{3}\int \left(t^3+2\right)^{-3}\left(t^3+2\right)'\,\mathrm{d}t\\ &= \tfrac{8}{3}\int \left(t^3+2\right)^{-3}\,\mathrm{d}\left(t^3+2\right)\\ &= \tfrac{8}{3}\left(-\tfrac{1}{2}\right)\left(t^3+2\right)^{-2}+C\\ &= -\frac{4}{3\left(t^3+2\right)^2}+C\end{aligned}$$

c $\quad \displaystyle\int \frac{1}{5x^2+16}\,\mathrm{d}x = \int \frac{1}{\left(x\sqrt{5}\right)^2+4^2}\,\mathrm{d}x$

Stel $t = x\sqrt{5}$, dan $\dfrac{\mathrm{d}t}{\mathrm{d}x} = \sqrt{5}$, zodat $\mathrm{d}x = \dfrac{1}{\sqrt{5}}\,\mathrm{d}t$.

Er volgt:

$$\begin{aligned}\int \frac{1}{5x^2+16}\,\mathrm{d}x &= \int \frac{1}{\left(x\sqrt{5}\right)^2+4^2}\,\mathrm{d}x\\ &= \frac{1}{\sqrt{5}}\int \frac{1}{t^2+4^2}\,\mathrm{d}t\\ &= \frac{1}{4}\frac{1}{\sqrt{5}}\arctan\left(\frac{t}{4}\right)+C\\ &= \tfrac{1}{20}\sqrt{5}\arctan\left(\frac{x\sqrt{5}}{4}\right)+C\end{aligned}$$

d

$$\begin{aligned}\int_0^2 4p\mathrm{e}^{3p^2-1}\,\mathrm{d}p &= \tfrac{4}{6}\int_0^2 \mathrm{e}^{3p^2-1}\left(3p^2-1\right)'\,\mathrm{d}p\\ &= \tfrac{2}{3}\int_{p=0}^{2} \mathrm{e}^{3p^2-1}\,\mathrm{d}\left(3p^2-1\right)\\ &= \tfrac{2}{3}\left[\mathrm{e}^{3p^2-1}\right]_{p=0}^{2} = \tfrac{2}{3}\left(\mathrm{e}^{11}-\mathrm{e}^{-1}\right)\end{aligned}$$

e

$$\int v\sin\left(3v^2-5\right)\mathrm{d}v=\tfrac{1}{6}\int\sin\left(3v^2-5\right)\mathrm{d}\left(3v^2-5\right)$$
$$=-\tfrac{1}{6}\cos\left(3v^2-5\right)+C$$

f

$$\int\frac{4}{x\ln^2 x}\mathrm{d}x=4\int\left(\ln x\right)^{-2}\mathrm{d}\ln x$$
$$=-\frac{4}{\ln x}+C$$

g

$$\int\frac{1}{\sqrt{4-x^2}}\mathrm{d}x=\int\frac{1}{\sqrt{2^2-x^2}}\mathrm{d}x=\arcsin\left(\tfrac{1}{2}x\right)+C$$

h

$$\int\frac{x}{\sqrt{4-x^2}}\mathrm{d}x=-\tfrac{1}{2}\int\left(4-x^2\right)^{-\frac{1}{2}}\mathrm{d}\left(4-x^2\right)$$
$$=-\tfrac{1}{2}(-2)\left(4-x^2\right)^{-\frac{1}{2}}+C$$
$$=-\sqrt{4-x^2}+C$$

Opgave 2

a

$$\int_0^2\frac{1}{x^2-x+1}\mathrm{d}x=\int_0^2\frac{1}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}\mathrm{d}x$$
$$=\int_{x=0}^{2}\frac{1}{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\mathrm{d}\left(x-\frac{1}{2}\right)$$
$$=\frac{2}{\sqrt{3}}\left[\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right]_{x=0}^{2}$$
$$=\frac{2}{\sqrt{3}}\left(\arctan\left(\frac{3}{\sqrt{3}}\right)-\arctan\left(-\frac{1}{\sqrt{3}}\right)\right)$$
$$=\tfrac{2}{3}\sqrt{3}\left(\tfrac{1}{3}\pi-\left(-\tfrac{1}{6}\pi\right)\right)=\tfrac{2}{3}\sqrt{3}\cdot\tfrac{1}{2}\pi=\tfrac{1}{3}\pi\sqrt{3}$$

b

$$\int_0^{\ln 3}\mathrm{e}^t\sqrt{\mathrm{e}^t+1}\,\mathrm{d}t=\int_{t=0}^{\ln 3}\left(\mathrm{e}^t+1\right)^{\frac{1}{2}}\mathrm{d}\left(\mathrm{e}^t+1\right)$$
$$=\left[\tfrac{2}{3}\left(\mathrm{e}^t+1\right)^{\frac{3}{2}}\right]_{t=0}^{\ln 3}$$
$$=\tfrac{2}{3}\cdot 4^{\frac{3}{2}}-\tfrac{2}{3}\cdot 2^{\frac{3}{2}}=\tfrac{16}{3}-\tfrac{4}{3}\sqrt{2}$$

c

$$\int \frac{7x}{5x^2+16}\,\mathrm{d}x = \tfrac{7}{10}\int \frac{\left(5x^2+16\right)'}{5x^2+16}\,\mathrm{d}x$$

$$= \tfrac{7}{10}\int \frac{1}{5x^2+16}\,\mathrm{d}\left(5x^2+16\right)$$

$$= \tfrac{7}{10}\ln\left(5x^2+16\right)+C$$

d Bepaal $\int 5w^3\sqrt{w^2-4}\,\mathrm{d}w$.

Stel $t = w^2-4$. Uit $t = w^2-4$ volgt $w^2 = t+4$ en $\mathrm{d}t = 2w\,\mathrm{d}w$, zodat $w\,\mathrm{d}w = \frac{1}{2}\mathrm{d}t$. We schrijven w^3 als w^2w. Na deze voorbereiding is de integraal te bepalen:

$$\int 5w^3\sqrt{w^2-4}\,\mathrm{d}w = 5\int w^2\sqrt{w^2-4}\,w\,\mathrm{d}w$$

$$= \frac{5}{2}\int (t+4)\sqrt{t}\,\mathrm{d}t$$

$$= \frac{5}{2}\int \left(t^{\frac{3}{2}}+4t^{\frac{1}{2}}\right)\mathrm{d}t$$

$$= t^2\sqrt{t}+\frac{20}{3}t\sqrt{t}+C$$

$$= \left(w^2-4\right)^2\sqrt{w^2-4}+\frac{20}{3}\left(w^2-4\right)\sqrt{w^2-4}+C$$

e Bereken $\int_1^3 \frac{1}{p\left(1+\sqrt{p}\right)}\,\mathrm{d}p$

Stel $x = \sqrt{p}$, hieruit volgt $p = x^2$ en $\mathrm{d}p = 2x\mathrm{d}x$.

Grenzen: $p=1 \rightarrow x=1$ en $p=3 \rightarrow x=\sqrt{3}$.

Uitvoeren van de substitutie levert op:

$$\int_1^3 \frac{1}{p\left(1+\sqrt{p}\right)}\,\mathrm{d}p = 2\int_1^{\sqrt{3}} \frac{x}{x^2(1+x)}\,\mathrm{d}x$$

$$= 2\int_1^{\sqrt{3}} \frac{1}{x(1+x)}\,\mathrm{d}x$$

Deze integraal is te berekenen met behulp van breuksplitsing.

Breuksplitsen leidt tot: $\dfrac{1}{x(x+1)} = \dfrac{A}{x}+\dfrac{B}{x+1}$. Hieruit volgt:

$$1 = A(1+x)+Bx$$

Kies $x=0$: $1=A \Rightarrow A=1$

Kies $x=-1$: $1=-B \Rightarrow B=-1$

Er volgt:

$$
\begin{aligned}
2\int_1^{\sqrt{3}} \frac{1}{x(1+x)}\,\mathrm{d}x &= 2\int_1^{\sqrt{3}} \frac{1}{x}\,\mathrm{d}x - 2\int_2^{\sqrt{3}} \frac{1}{x+1}\,\mathrm{d}x \\
&= 2\int_1^{\sqrt{3}} \frac{1}{x}\,\mathrm{d}x - 2\int_2^{\sqrt{3}} \frac{1}{x+1}\,\mathrm{d}(x+1) \\
&= 2\left[\ln|x|\right]_1^{\sqrt{3}} - 2\left[\ln|x+1|\right]_1^{\sqrt{3}} \\
&= 2\ln\left(\sqrt{3}\right) - 2\ln\left(1+\sqrt{3}\right) + 2\ln(2) \\
&= \ln(3) - 2\ln\left(1+\sqrt{3}\right) + 2\ln(2)
\end{aligned}
$$

f Bepaal $\int \sin(5t)\cos(3t)\,\mathrm{d}t$

$$
\begin{aligned}
\int \sin(5t)\cos(3t)\,\mathrm{d}t &= \tfrac{1}{2}\int \left(\sin(8t) + \sin(2t)\right)\mathrm{d}t \\
&= \tfrac{1}{2}\int \sin(8t)\,\mathrm{d}t + \tfrac{1}{2}\int \sin(2t)\,\mathrm{d}t \\
&= \tfrac{1}{16}\int \sin(8t)\,\mathrm{d}(8t) + \tfrac{1}{4}\int \sin(2t)\,\mathrm{d}(2t) \\
&= -\tfrac{1}{16}\cos(8t) - \tfrac{1}{4}\cos(2t) + C
\end{aligned}
$$

g Bepaal $\int 5t^3 \sin^2\left(2t^4\right)\mathrm{d}t$

Gebruik $\sin^2(x) = \tfrac{1}{2}\left(1-\cos(2x)\right)$, zodat $\sin^2\left(2t^4\right) = \tfrac{1}{2}\left(1-\cos\left(4t^4\right)\right)$

Er volgt:

$$
\begin{aligned}
\int 5t^3 \sin^2\left(2t^4\right)\mathrm{d}t &= \tfrac{5}{2}\int t^3\left(1-\cos\left(4t^4\right)\right)\mathrm{d}t \\
&= \tfrac{5}{2}\int t^3\,\mathrm{d}t - \tfrac{5}{2}\int t^3\cos\left(4t^4\right)\mathrm{d}t \\
&= \tfrac{5}{2}\int t^3\,\mathrm{d}t - \tfrac{5}{2}\cdot\tfrac{1}{16}\int \cos\left(4t^4\right)\mathrm{d}\left(4t^4\right) \\
&= \tfrac{5}{8}t^4 - \tfrac{5}{32}\sin\left(4t^4\right) + C
\end{aligned}
$$

h Bereken $\displaystyle\int_4^9 \frac{1}{(y-1)\sqrt{y}}\,\mathrm{d}y$.

Stel $p = \sqrt{y}$, hieruit volgt $y = p^2$ en $\mathrm{d}y = 2p\,\mathrm{d}p$.

Grenzen: $y = 4 \rightarrow p = 2$ en $y = 9 \rightarrow p = 3$.

Uitvoeren van de substitutie levert op:

$$
\begin{aligned}
\int_4^9 \frac{1}{(y-1)\sqrt{y}}\,\mathrm{d}y &= 2\int_2^3 \frac{p}{\left(p^2-1\right)p}\,\mathrm{d}p \\
&= 2\int_2^3 \frac{1}{p^2-1}\,\mathrm{d}p
\end{aligned}
$$

Deze integraal is te berekenen met behulp van breuksplitsing.

De noemer is te ontbinden als $p^2 - 1 = (p-1)(p+1)$.

Breuksplitsen leidt tot: $\dfrac{2}{(p-1)(p+1)}=\dfrac{A}{p-1}+\dfrac{B}{p+1}$. Hieruit volgt:

$$2=A(p+1)+B(p-1)$$

Kies $p=1$: $2=2A\Rightarrow A=1$

Kies $p=-1$: $2=-2B\Rightarrow B=-1$

Er volgt:

$$\begin{aligned}2\int_2^3\frac{1}{p^2-1}\,\mathrm{d}p&=\int_2^3\frac{1}{p-1}\,\mathrm{d}p-\int_2^3\frac{1}{p+1}\,\mathrm{d}p\\&=\int_2^3\frac{1}{p-1}\,\mathrm{d}(p-1)-\int_2^3\frac{1}{p+1}\,\mathrm{d}(p+1)\\&=\left[\ln|p-1|\right]_2^3-\left[\ln|p+1|\right]_2^3\\&=\ln(2)-\ln(4)+\ln(3)\\&=\ln\left(\tfrac{3}{2}\right)\end{aligned}$$

Opgave 3

a Bereken $\displaystyle\int_0^{\frac14\pi}\frac{p}{\cos^2 p}\,\mathrm{d}p$.

$$\begin{aligned}\int_0^{\frac14\pi}\frac{p}{\cos^2 p}\,\mathrm{d}p&=\int_0^{\frac14\pi}p\,\mathrm{d}\tan p\\&=\left[p\tan p\right]_0^{\frac14\pi}-\int_0^{\frac14\pi}\tan p\,\mathrm{d}p\\&=\tfrac14\pi-\int_0^{\frac14\pi}\frac{\sin p}{\cos p}\,\mathrm{d}p\\&=\tfrac14\pi+\int_0^{\frac14\pi}\frac{1}{\cos p}\,\mathrm{d}\cos p\\&=\tfrac14\pi+\left[\ln|\cos p|\right]_0^{\frac14\pi}\\&=\tfrac14\pi+\ln\left|\cos\tfrac14\pi\right|-\ln|\cos 0|\\&=\tfrac14\pi+\ln\tfrac12\sqrt{2}\\&=\tfrac14\pi-\tfrac12\ln 2\end{aligned}$$

b Bepaal $\displaystyle\int\frac{1}{x^3+5x^2}\,\mathrm{d}x$.

Oplossing:

De noemer is te ontbinden als $x^3+5x^2=x^2(x+5)$.

Breuksplitsen leidt tot: $\dfrac{1}{x^2\left(x+5\right)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+5}$. Hieruit volgt:

$$1=Ax\left(x+2\right)+B\left(x+5\right)+Cx^2$$

Kies $x=0$: $1=5B\Rightarrow B=\frac{1}{5}$

Kies $x=-5$: $1=25C\Rightarrow C=\frac{1}{25}$

Coëfficiënt van x^2: $0=A+C\Rightarrow A=-\frac{1}{25}$.

Er volgt:

$$\int\frac{1}{x^3+5x^2}\mathrm{d}x=-\tfrac{1}{25}\int\frac{1}{x}\mathrm{d}x+\tfrac{1}{5}\int\frac{1}{x^2}\mathrm{d}x+\tfrac{1}{25}\int\frac{1}{x+5}\mathrm{d}x$$

$$=-\tfrac{1}{25}\ln\left|x\right|-\frac{1}{5x}+\tfrac{1}{25}\ln\left|x+5\right|+C$$

c Bepaal $\displaystyle\int\frac{5x}{x^2+2x+5}\mathrm{d}x$

$$\int\frac{5x}{x^2+2x+5}\mathrm{d}x=\tfrac{5}{2}\int\frac{2x+2}{x^2+2x+5}\mathrm{d}x-5\int\frac{1}{x^2+2x+5}\mathrm{d}x$$

$$=\tfrac{5}{2}\int\frac{1}{x^2+2x+5}\mathrm{d}\left(x^2+2x+5\right)-5\int\frac{1}{\left(x+1\right)^2+4}\mathrm{d}\left(\mathrm{x}+1\right)$$

$$=\tfrac{5}{2}\ln\left(x^2+2x+5\right)-\tfrac{5}{2}\arctan\left(\frac{x+1}{2}\right)+C$$

d Bereken $\displaystyle\int_1^2\frac{x+1}{x^3+6x^2+9x}\mathrm{d}x$

De noemer is te ontbinden als $x^3+6x^2+9x=x\left(x+3\right)^2$.

Breuksplitsen leidt tot: $\dfrac{x+1}{x\left(x+3\right)^3}=\dfrac{A}{x}+\dfrac{B}{x+3}+\dfrac{C}{\left(x+3\right)^2}$. Hieruit volgt:

$$x+1=A\left(x+3\right)^2+Bx\left(x+3\right)+Cx$$

Kies $x=-3$: $-2=-3C\Rightarrow C=\frac{2}{3}$

Kies $x=0$: $1=9A\Rightarrow A=\frac{1}{9}$

Coëfficiënt van x^2: $0=A+B\Rightarrow B=-\frac{1}{9}$.

Er volgt:

$$\int_1^2 \frac{x+1}{x^3+6x^2+9x}\,\mathrm{d}x = \tfrac{1}{9}\int_1^2 \frac{1}{x}\,\mathrm{d}x - \tfrac{1}{9}\int_2^3 \frac{1}{x+3}\,\mathrm{d}x + \tfrac{2}{3}\int_2^3 \frac{1}{(x+3)^2}\,\mathrm{d}x$$
$$= \tfrac{1}{9}\int_1^2 \frac{1}{x}\,\mathrm{d}x - \tfrac{1}{9}\int_2^3 \frac{1}{x+3}\,\mathrm{d}(x+3) + \tfrac{2}{3}\int_2^3 (x+3)^{-2}\,\mathrm{d}(x+3)$$
$$= \tfrac{1}{9}\left[\ln|x|\right]_1^2 - \tfrac{1}{9}\left[\ln|x+3|\right]_1^2 - \tfrac{2}{3}\left[\frac{1}{x+3}\right]_1^2$$
$$= \tfrac{1}{2}\ln(2) - \tfrac{1}{9}\ln(5) + \tfrac{1}{9}\ln(4) - \tfrac{2}{3}\left(\tfrac{1}{5} - \tfrac{1}{4}\right)$$
$$= \tfrac{1}{3}\ln(2) - \tfrac{1}{9}\ln(5) + \tfrac{1}{30}$$

e Bepaal $\int \arctan(3x)\,\mathrm{d}x$

We voeren direct partiële integratie uit volgens de formule

$$\int f(x)\,g'(x)\,\mathrm{d}x = f(x)\,g(x) - \int g(x)\,f'(x)\,\mathrm{d}x,$$

waarbij $f(x) = \arctan 3x$ en $g(x) = x$:

$$\int \arctan(3x)\,\mathrm{d}x = x\arctan(3x) - \int x\,\mathrm{d}\left(\arctan(3x)\right) \quad \left(\mathrm{d}(\arctan 3x) = \frac{3}{1+9x^2}\,\mathrm{d}x\right)$$
$$= x\arctan(3x) - \int \frac{3x}{1+9x^2}\,\mathrm{d}x$$
$$= x\arctan(3x) - \tfrac{1}{6}\int \frac{\left(1+9x^2\right)'}{1+9x^2}\,\mathrm{d}x$$
$$= x\arctan(3x) - \tfrac{1}{6}\int \frac{1}{1+9x^2}\,\mathrm{d}\left(1+9x^2\right)$$
$$= x\arctan(3x) - \tfrac{1}{6}\ln\left(1+9x^2\right) + C$$

f Bereken $\int_0^2 z\cdot 3^{4z}\,\mathrm{dz}$.

We moeten 3^{4z} achter het d-teken brengen. Om 3^{4z} achter het d-teken te kunnen brengen, bepalen we eerst $\int 3^{4z}\,\mathrm{dz}$:

$\int 3^{4z}\,\mathrm{dz} = \tfrac{1}{4}\int 3^{4z}\,\mathrm{d}(4z) = \frac{1}{4\ln 3}4^{3z} + C$, zodat $3^{4z}\mathrm{dz} = \mathrm{d}\left(\tfrac{1}{4\ln 3}3^{4z}\right) = \tfrac{1}{4\ln 3}\mathrm{d}\left(3^{4z}\right)$.

Er volgt:

$$\int_0^2 z\cdot 3^{4z}\,\mathrm{d}z = \frac{1}{4\ln 3}\int_0^2 z\,\mathrm{d}\left(3^{4z}\right)$$

$$= \left[\frac{1}{4\ln 3}z\cdot 3^{4z}\right]_0^2 - \frac{1}{4\ln 3}\int_0^2 3^{4z}\,\mathrm{d}z$$

$$= \frac{2\cdot 3^8}{4\ln 3} - \left(\frac{1}{4\ln 3}\right)^2\left[3^{4z}\right]_0^2$$

$$= \frac{3^8}{2\ln 3} - \frac{1}{16(\ln 3)^2}\left(3^8 - 1\right)$$

$$= \frac{6561}{2\ln 3} - \frac{410}{(\ln 3)^2}$$

$$= \frac{6561\cdot \ln 3410 - 820}{2(\ln 3)^2}$$

g Bereken $\int_5^9 \frac{x+8}{x^3-6x^2+8x}\,\mathrm{d}x$.

Uitwerking:

De noemer is te ontbinden als $x^3-6x^2+8x = x(x-4)(x-2)$.

Breuksplitsen leidt tot: $\frac{x+8}{x(x-4)(x-2)} = \frac{A}{x}+\frac{B}{x-4}+\frac{C}{x-2}$. Hieruit volgt:

$$x+8 = A(x-4)(x-2)+Bx(x-2)+Cx(x-4)$$

Kies $x=4$: $12 = 8B \Rightarrow B = \frac{3}{2}$

Kies $x=2$: $10 = -4C \Rightarrow C = -\frac{10}{4} = -\frac{5}{2}$

Kies $x=0$: $8 = 8A \Rightarrow A = 1$

Er volgt:

$$\int_5^9 \frac{x+8}{x^3-6x^2+8x}\,\mathrm{d}x = \int_5^9 \frac{1}{x}\,\mathrm{d}x + \frac{3}{2}\int_5^9 \frac{1}{x-4}\,\mathrm{d}x - \frac{5}{2}\int_5^9 \frac{1}{x-2}\,\mathrm{d}x$$

$$= \int_5^9 \frac{1}{x}\,\mathrm{d}x + \frac{3}{2}\int_5^9 \frac{1}{x-4}\,\mathrm{d}(x-4) - \frac{5}{2}\int_5^9 \frac{1}{x-2}\,\mathrm{d}(x-2)$$

$$= \left[\ln|x|\right]_5^9 + \frac{3}{2}\left[\ln|x-4|\right]_5^9 - \frac{5}{2}\left[\ln|x-2|\right]_5^9$$

$$= \ln(9)-\ln(5)+\frac{3}{2}\ln(5)-\frac{5}{2}\ln(7)+\frac{5}{2}\ln(3)$$

$$= \frac{9}{2}\ln(3)+\frac{1}{2}\ln(5)-\frac{5}{2}\ln(7)$$

h Bepaal $\int 8t^2\sin(4t)\,\mathrm{d}t$.

We moeten de factor $\sin(4t)$ achter het d-teken brengen. We bepalen eerst apart $\int \sin(4t)\,\mathrm{d}t$:

$\int \sin(4t)\,\mathrm{d}t = \frac{1}{4}\int \sin(4t)\,\mathrm{d}(4t) = -\frac{1}{4}\cos(4t) + C$, zodat $\sin(4t)\,\mathrm{d}t = -\frac{1}{4}\mathrm{d}(\cos(4t))$.

Er volgt:

$$\begin{aligned}\int 8t^2\sin(4t)\,\mathrm{d}t &= -2\int t^2\,\mathrm{d}(\cos(4t))\\ &= -2t^2\cos(4t) + 2\int \cos(4t)\,\mathrm{d}(t^2)\\ &= -2t^2\cos(4t) + 4\int t\cos(4t)\,\mathrm{d}t\\ &= -2t^2\cos(4t) + \int t\,\mathrm{d}(\sin(4t))\\ &= -2t^2\cos(4t) + t\sin(4t) - \int \sin(4t)\,\mathrm{d}t\\ &= -2t^2\cos(4t) + t\sin(4t) + \frac{1}{4}\cos(4t) + C\end{aligned}$$

Opgave 4

a Noem de integrand $f(x) = \mathrm{e}^{\sin x}$

Deelintervallengte $h = \frac{3-1}{4} = \frac{1}{2}$. Tabel van functiewaarden:

x	$x_0 = 1$	$x_1 = 1{,}5$	$x_2 = 2$	$x_3 = 2{,}5$	$x_4 = 3$
$f(x)$	$\mathrm{e}^{\sin 1}$	$\mathrm{e}^{\sin 1{,}5}$	$\mathrm{e}^{\sin 2}$	$\mathrm{e}^{\sin 2{,}5}$	$\mathrm{e}^{\sin 3}$

We vinden:

$$\begin{aligned}T_4 &= \frac{1}{2}h\{f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\}\\ &= \frac{1}{4}\left(\mathrm{e}^{\sin 1} + 2\cdot\mathrm{e}^{\sin 1{,}5} + 2\cdot\mathrm{e}^{\sin 2} + 2\cdot\mathrm{e}^{\sin 2{,}5} + \mathrm{e}^{\sin 3}\right)\\ &= 4.374532783\end{aligned}$$

b

$$\begin{aligned}S_4 &= \frac{1}{3}h\{f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\}\\ &= \frac{1}{6}\left(\mathrm{e}^{\sin 1} + 4\cdot\mathrm{e}^{\sin 1{,}5} + 2\cdot\mathrm{e}^{\sin 2} + 4\cdot\mathrm{e}^{\sin 2{,}5} + \mathrm{e}^{\sin 3}\right)\\ &= 4.426627858\end{aligned}$$

c Als we $I = \int_1^3 \mathrm{e}^{\sin x}\,\mathrm{d}x$ benaderen met de trapeziumregel toegepast op 4 deelintervallen,

dan geldt voor de maximale fout in absolute waarde $|E_T| \le \frac{(3-1)^3}{12\cdot 4^2}\max\limits_{1\le x\le 3}|f''(x)|$

De tweede afgeleide van $f(x) = \mathrm{e}^{\sin x}$: $f''(x) = -\sin(x)\mathrm{e}^{\sin x} + \cos^2(x)\mathrm{e}^{\sin x}$.

Voor het berekenen van het gevraagde maximum berekenen we de derde afgeleide:

$$\begin{aligned}f'''(x) &= -\cos(x)\mathrm{e}^{\sin x} - 3\sin(x)\cos(x)\mathrm{e}^{\sin x} + \cos^3(x)\mathrm{e}^{\sin x}\\ &= \cos(x)\mathrm{e}^{\sin x}\left(-1 - 3\sin(x) + \cos^2(x)\right)\end{aligned}$$

$$f'''(x) = 0 \Leftrightarrow \cos(x) = 0 \vee \cos^2(x) - 1 - 3\sin(x) = 0$$

Opgelost wordt: $\cos(x) = 0$ en $\cos^2(x) - 1 - 3\sin(x) = 0$:

$$\cos(x) = 0 \Rightarrow x = \frac{1}{2}\pi + k\pi, \text{ met } k \in \mathbb{Z}$$

$$\cos^2(x) - 1 - 3\sin(x) = 0 \Rightarrow 1 - \sin^2 x - 1 - 3\sin(x) = 0$$
$$\Rightarrow \sin^2 x + 3\sin(x) = 0 \Rightarrow \sin x(\sin x + 3) = 0$$
$$\Rightarrow \sin x = 0 \Rightarrow x = k \cdot \pi, \text{ met } k \in \mathbb{Z}$$

Het maximum van $|f''(x)|$ op het interval $[1,3]$ wordt bereikt voor $x = \frac{1}{2}\pi$ of op de rand van het interval.
Vergelijken van de waarden geeft:
$f''(1) = -1,2748$, $f''(3) = 0,9661$ en $f''\left(\frac{1}{2}\pi\right) = -\mathrm{e}$,

zodat $\max\limits_{1 \le x \le 3} |f''(x)| = \mathrm{e}$.

Er volgt: $|E_T| \le \dfrac{(3-1)^3 \cdot \mathrm{e}}{12 \cdot 4^2} = \frac{1}{24}\mathrm{e}$

d Er moet gelden

$$|E_T| \le \frac{2^3}{12 \cdot n^2} \max_{1 \le x \le 3} |f''(x)| \underset{\text{eis}}{<} \frac{1}{2} \cdot 10^{-6}$$

Er volgt, met $\max\limits_{1 \le x \le 3} |f''(x)| = \mathrm{e}$:

$$\frac{2^3}{12 \cdot n^2} \cdot \mathrm{e} < \frac{1}{2} \cdot 10^{-6} \Rightarrow n > \sqrt{\frac{2^4 \cdot 10^6 \cdot \mathrm{e}}{12}} = 1903,8$$

Het aantal deelintervallen moeten we minimaal $n = 1904$ nemen om er zeker van te zijn dat I in 6 decimalen nauwkeurig benaderd wordt.

Opgave 5

a

$$\int_0^3 \frac{\mathrm{e}^{\frac{1}{x}}}{x^2} \mathrm{d}x = \lim_{p \downarrow 0} \int_p^3 \frac{\mathrm{e}^{\frac{1}{x}}}{x^2} \mathrm{d}x$$

$$= \lim_{p \downarrow 0} - \int_p^3 \frac{\mathrm{e}^{\frac{1}{x}}}{x^2} \mathrm{d}\left(\frac{1}{x}\right)$$

$$= \lim_{p \downarrow 0} - \left[\mathrm{e}^{\frac{1}{x}}\right]_p^3 = \lim_{p \downarrow 0} \left(-\mathrm{e}^{\frac{1}{3}} + \mathrm{e}^{\frac{1}{p}}\right) = \infty$$

De integraal divergeert.

b

$$\int_0^3 \frac{e^{-\frac{1}{x}}}{x^2}\,dx = \lim_{p\downarrow 0}\int_p^3 \frac{e^{-\frac{1}{x}}}{x^2}\,dx$$

$$= \lim_{p\downarrow 0}\int_p^3 \frac{e^{-\frac{1}{x}}}{x^2}\,d\left(-\frac{1}{x}\right)$$

$$= \lim_{p\downarrow 0}\left[e^{-\frac{1}{x}}\right]_p^3$$

$$= \lim_{p\downarrow 0}\left(e^{-\frac{1}{3}} - e^{-\frac{1}{p}}\right) = e^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{e}}$$

De integraal convergeert naar $e^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{e}}$.

c $\quad \int_{-\infty}^{-2} \frac{4}{x-1}\,dx = \lim_{b\to-\infty}\int_b^{-2} \frac{4}{x-1}\,dx = \lim_{b\to-\infty}\left[4\ln|x-1|\right]_b^{-2} = \lim_{b\to-\infty}\left(4\ln 3 - 4\ln|b-1|\right) = -\infty$

d

$$\int_{-\infty}^{\infty} \frac{e^x}{e^{2x}+1}\,dx = \lim_{s\to-\infty}\int_s^0 \frac{1}{\left(e^x\right)^2+1}\,d\left(e^x\right) + \lim_{t\to\infty}\int_0^t \frac{1}{\left(e^x\right)^2+1}\,d\left(e^x\right)$$

$$= \lim_{s\to-\infty}\left[\arctan\left(e^x\right)\right]_s^0 + \lim_{t\to\infty}\left[\arctan\left(e^x\right)\right]_0^t$$

$$= \lim_{s\to-\infty}\left(\arctan 1 - \arctan\left(e^s\right)\right) + \lim_{t\to\infty}\left(\arctan\left(e^t\right) - \arctan 1\right)$$

$$= \tfrac{1}{4}\pi - 0 + \tfrac{1}{2}\pi - \tfrac{1}{4}\pi = \tfrac{1}{2}\pi$$

De integraal convergeert naar $\frac{1}{2}\pi$.

Opgave 6

De oppervlakte van het meer is:

$O = \int_0^{240} b(x)\,dx$, met $b = b(x)$ de breedte van het meer op de plaats x.

a

$$T_{12} = \tfrac{1}{2}\cdot 20\left\{b(0) + 2\cdot b(20) + 2\cdot b(40) + 2\cdot b(60) + \cdots + 2\cdot b(200) + 2\cdot b(220) + b(240)\right\}$$

$$= 10\left\{0 + 2\cdot 80 + 2\cdot 84 + 2\cdot 72 + 2\cdot 51 + 2\cdot 42 + 2\cdot 47 + 2\cdot 59 + 2\cdot 81 + 2\cdot 115 + 2\cdot 124 + 2\cdot 86 + 0\right\}$$

$$= 16820 \text{ m}^2$$

b

$$S_{12} = \tfrac{1}{3}\cdot 20\left\{b(0) + 4\cdot b(20) + 2\cdot b(40) + 4\cdot b(60) + \cdots + 2\cdot b(200) + 4\cdot b(220) + b(240)\right\}$$

$$= \tfrac{1}{3}\cdot 20\left\{0 + 4\cdot 80 + 2\cdot 84 + 4\cdot 72 + 2\cdot 51 + 4\cdot 42 + 2\cdot 47 + 4\cdot 59 + 2\cdot 81 + 4\cdot 115 + 2\cdot 124 + 4\cdot 86 + 0\right\}$$

$$= 17266{,}667 \text{ m}^2$$